3. VIBRATION ANALYSIS

It has been stated that the reason of the failure of electronic equipment due to vibration is the stress induced in the mounting system due to the relative motion between the printed circuit board and the electronic component. The deformation of the PCB is the response to the vibration excitation provided by its enviroment, usually a machine. It is a function then of both the excitation and its own structural properties. Before attempting to reduce the response, it is important to understand the vibration process.

3.1 SINGLE DEGREE OF FREEDOM MODEL

The simplest but still useful model used for vibration analysis is a single degree of freedom mass system with stiffness and damping, as depicted in the figure. In the case of electronic equipment the mass will usually be the PCB, and the base the machine or electronic box to which it is fixed.

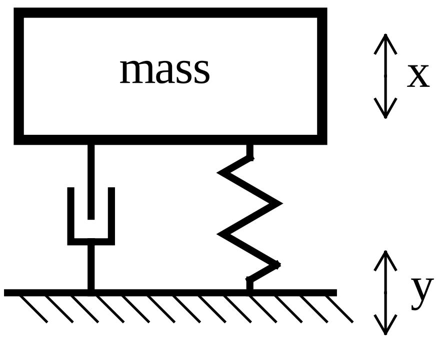

Fig. 3.1.- Single degree of freedom model.

It is usual to assume that the base is static, but since the case of interest here is clearly the opposite, two degrees of freedom will be considered for the global system, which are y(t), the absolute motion of the base, and $x_1(t)$, the motion of the mass, in this case the PCB.

Assuming there is no external excitation appart from the displacement of the base, the equation of motion of the mass is

$$m\,\ddot{x} + c\,(\dot{x} - \dot{y}) + k\,(x - y) = 0 \qquad (3\text{-}1)$$

where m is the mass, c is the damping and k the stiffness. Introducing the relative deflection $z = x - y$

$$m\,\ddot{z} + c\,\dot{z} + k\,\dot{z} = -\,m\,\ddot{y} \qquad (3\text{-}2)$$

When divided by m the equation can be expressed as

$$\ddot{z} + 2\,\Omega\,\zeta\,\dot{z} + \Omega^2\,\dot{z} = -\,\ddot{y} \qquad (3\text{-}3)$$

where $\Omega = \dfrac{k}{m}$ is the natural frequency and $\zeta = \dfrac{c}{2\,\Omega\,m}$ the loss factor.

For a sinusoidal base motion with equations of motion

$$y\,(t) = Y \sin \omega t \qquad\qquad \ddot{y}\,(t) = -\,\omega^2\;Y \sin \omega t$$

the magnitude of the relative deflection z, as a function of the frequency of excitation, is

$$Z(\omega) = \left| z\,(\omega) \right| = \frac{\omega^2\;Y}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\,\omega^2\;\Omega^2\;\zeta^2}} \qquad (3\text{-}4)$$

This is the typical response for a single degree of fredom system, and it is shown in Fig. 3.2. It is clear that the highest values will appear when the frequency of the excitation is similar to the natural frequency (marked with a line in the figure).

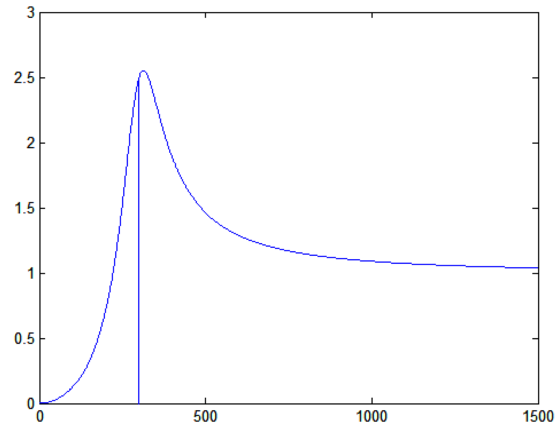

Fig. 3.2.- Typical response curve for a s.d.f. system.

It is useful, however, to focus only on the contribution of the system. The transfer function H is the ratio between the maximum deflection and the maximum input force for a given excitation frequency

$$H\,(\omega) = \frac{1}{m\,\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\,\omega^2\;\Omega^2\;\zeta^2}} \qquad (3\text{-}5)$$

The behaviour is similar to the one found for the relative deflection, specially for excitations near the natural frequency of the system, which again are noticeably amplified, as can be seen in Fig. 3.3. In fact, the maximum of the transfer function (marked in the figure) is found in

$$\omega^* = \Omega_1 \sqrt{1 - 2\,\zeta_1^{\;2}} \qquad (3\text{-}6)$$

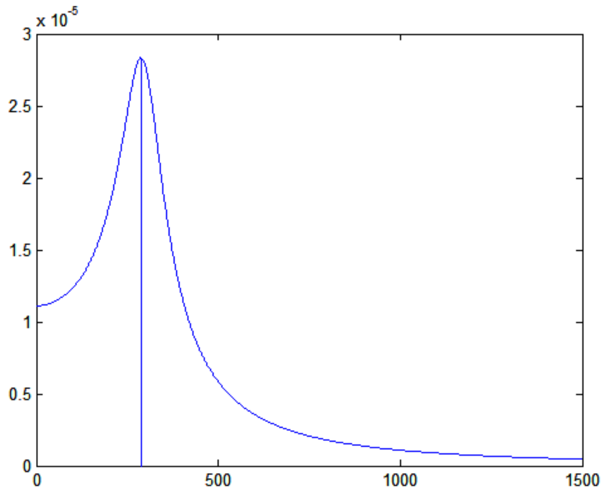

Fig. 3.3.- Typical transfer function of a s.d.f. system.

Finally, it is possible to study the transmissibility of the system, T, defined as the ratio between the maximum output force F to the maximum input force P, which in this case is

$$P = \max(m\,\ddot{y}) = m\,\ddot{Y} = m\,\omega^2\,Y \qquad (3\text{-}7)$$

The output force is the sum of the spring force and the damping force. As both forces have a 90º phase angle, the total force is

$$F = Z\sqrt{k^2 + c^2\,\Omega^2} = \frac{\omega^2\,Y\sqrt{k^2 + c^2\,\Omega^2}}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\,\omega^2\,\Omega^2\,\zeta^2}} \qquad (3\text{-}8)$$

Using equations (3-7) and (3-8) the transmissibility is then

$$T = \frac{F}{P} = \frac{\sqrt{k^2 + c^2\,\Omega^2}}{m\,\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\,\omega^2\,\Omega^2\,\zeta^2}} = \frac{\sqrt{\Omega^4 + 4\,\omega^2\,\Omega^2\,\zeta^2}}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\,\omega^2\,\Omega^2\,\zeta^2}} \qquad (3\text{-}9)$$

The transmissibility represents also the ration between the absolute output acceleration to the input acceleration. The equilibrium of forces in the mass produces the relation

$$m\,\ddot{x} + c\,(\dot{x} - \dot{y}) + k\,(x - y) = 0 \qquad (3\text{-}10)$$

$$m\,\ddot{x} = -\,c\,\dot{z} - k\,z \qquad (3\text{-}11)$$

This way the output force, which as said before is the sum of both the spring and damping force, can be expressed also as

$$F = \max(m\,\ddot{x}) = m\,\ddot{X} = m\,\omega^2\,X \qquad (3\text{-}12)$$

So introducing both equations (3-7) and (3-12) in (3-9)

$$T = \frac{F}{P} = \frac{\ddot{X}}{\ddot{Y}}$$

The relation applies also to displacement and velocity, due to the characteristic of the sinosoidal vibration. The behaviour of the function is again similar to the previous ones, as seen in Fig. 3.4.

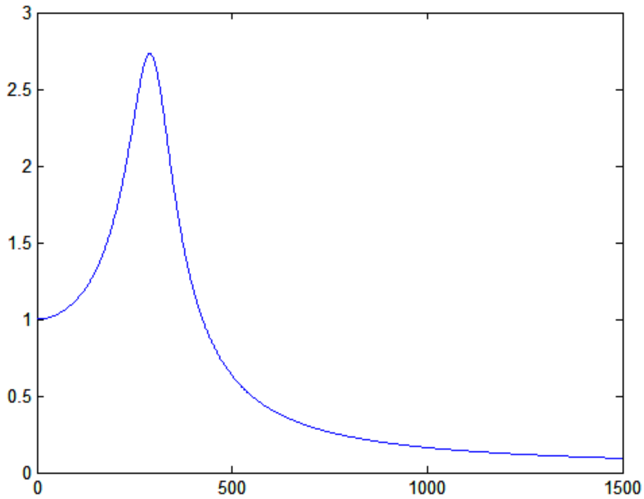

Fig. 3.4.- Typical transmissibility of a s.d.f. system

The peak in the natural frequency range, due to the $\left(\Omega^2 - \omega^2\right)$ factor in the denominator, is the resonance phenomenon, in which an excitation is amplified by the own system, sometimes leading to catastrophic results. On the contrary, when the excitation frequency is very different from the resonant frequency, the system tends to atenuate the vibration. It is essential then to assure that the natural frequency is far from the range of excitation frequencies anticipated. In practice this usually means to try to have a resonant frequency in the PCB as high as possible. This will be stated more clearly in OCTAVE RULE.

3.2 MULTYBODY SYSTEMS

The vibration is not produced directly on the PCB, but transmitted through the tool from its original point. During this process the excitation can be amplified or reduced according to the dynamic response of the different elements involved, which is then another factor to be taken into account. Again, a simple model will be used for understanding the phenomenon and performing rough estimations.

The idea is to extend the spring-mass system, adding as masses as necessary to model the different parts of the real system. The model shown in Fig. 3.5 is the traditionally used for a PCB (mass 2) and its electronic box (mass 1). The vibration first affects the enclosure and then is transmitted to the board. The equations of motion are

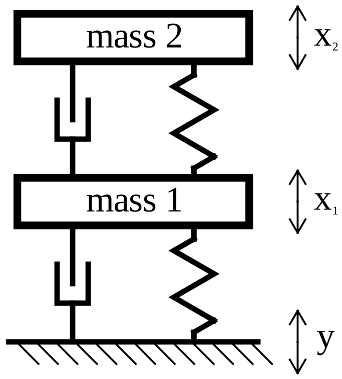

Fig. 3.5.- Two degrees of freedom system.

$$m_2\ \ddot{x}_2 + c_2\ (\dot{x}_2 - \dot{x}_1) + k_2\ (x_2 - x_1) = 0 \qquad (3\text{-}13)$$

$$m_1\ \ddot{x}_1 + c_1\ (\dot{x}_1 - \dot{y}) + k_1\ (x_1 - y) - c_2\ (\dot{x}_2 - \dot{x}_1) - k_2\ (x_2 - x_1) = 0 \qquad (3\text{-}14)$$

Introducing the relative deflections $z_1 = x_1 - y$ and $z_2 = x_2 - x_1$

$$m_2\ \ddot{z}_2 + c_2\ \dot{z}_2 + k_2\ z_2 = -\ m_2\ \ddot{x}_1 \qquad (3\text{-}15)$$

$$m_1\ \ddot{z}_1 + c_1\ \dot{z}_1 + k_1\ z_1 = -\ m_1\ \ddot{y} - m_2\ \ddot{x}_2 \qquad (3\text{-}16)$$

The first equation is the same that in the single mass-spring model. The second one is much more complex, since the responses of both masses are involved. However it is usually simplified assuming that the mass of the second system (which normally is the PCB) is very small, so it can be neglected from the second equation

$$m_2\ \ddot{z}_2 + c_2\ \dot{z}_2 + k_2\ z_2 = -\ m_2\ \ddot{x}_1 \qquad (3\text{-}17)$$

$$m_1\ \ddot{z}_1 + c_1\ \dot{z}_1 + k_1\ z_1 = -\ m_1\ \ddot{y} \qquad (3\text{-}18)$$

The assumption that the mass of the PCB is small enough to not influence the primary system will be used during this document unless the opposite it is explicitly mentioned. The system is then composed of two mass-spring systems in serie. The input for the first mass is the base acceleration, and its output is the input for the second mass. Therefore, the response of the second system will be dependant of both transmissibilities

$$\ddot{x}_2 = T_2\ \ddot{x}_1 = T_2\ T_1\ \ddot{y} \qquad (3\text{-}19)$$

where

$$T_1(\omega) = \sqrt{\frac{{\Omega_1}^4 + 4\omega^2\ {\Omega_1}^2\ {\zeta_1}^2}{\left({\Omega_1}^2 - \omega^2\right)^2 + 4\omega^2\ {\Omega_1}^2\ {\zeta_1}^2}} \qquad (3\text{-}20)$$

$$T_2(\omega) = \sqrt{\frac{\Omega_2{}^4 + 4\omega^2\,\Omega_2{}^2\,\zeta_2{}^2}{\left(\Omega_2{}^2 - \omega^2\right)^2 + 4\omega^2\,\Omega_2{}^2\,\zeta_2{}^2}} \qquad (3\text{-}21)$$

Another interesting function is the relative transmissibility of the second subsystem, which relates its relative deflection to the absolute deflection of the first mass. The assumption of sinusoidal excitation leads to

$$Z_2(\omega) = \frac{\ddot{X}_1}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\,\omega^2\,\Omega^2\,\zeta^2}} = \frac{\omega^2\,X_1}{\sqrt{\left(\Omega^2 - \omega^2\right)^2 + 4\,\omega^2\,\Omega^2\,\zeta^2}} \qquad (3\text{-}22)$$

The equation of the function is therefore

$$T_2^{rel}(\omega) = \frac{\omega^2}{\sqrt{\left(\Omega_2{}^2 - \omega^2\right)^2 + 4\omega^2\,\Omega_2{}^2\,\zeta_2{}^2}} \qquad (3\text{-}23)$$

and the relative deflection of the second mass can be expressed as

$$Z_2 = T_2^{rel}\,X_1 = T_2^{rel}\,\frac{\ddot{X}_1}{\omega^2} = \frac{1}{\omega^2}\,T_2^{rel}\,T_1\,\ddot{Y} \qquad (3\text{-}24)$$

In systems with more than one degree of freedom the concept of eigenmodes appears. Now there is a natural frequency and a vibration mode for each of the subsystems, both of them solution to the generalized eigen values problem associated to the equation of motion of the system. The later is a vector which states the relation between the different degrees of freedom. If the structure is excited with vibration at one of the resonance frequencies, the response will have the shape of the mode, with the actual displacements depending on the magnitude of the excitation. When a different frequency is used, the response will be a linear combination of the eigen modes, as they form a base in the vectorial space of the structure degrees of freedom.

In reality, a structure is an infitine degree of freedom system, with infinite resonance frequencies and vibration modes, although the usual procedure is to discretize it into a finite number.

3.3 BOARD MODEL

The single degree of freedom model of the PCB is useful to obtain a general understanding of the vibration process and to achieve results as a first order approximation. It is however obvious that a more realistic model will be necessary during some parts of the analysis. It should be able to provide more accurate results, but still through a relatively fast and simple analysis.

A finite element model using classical plate theory will be use. The foundations of both FEM and plate theory can be found in several text books. It is still important to discuss the concrete model used, that is, the different properties and parameters.

Experimental testing shows that a suitable modelling consists of using a plate with same structural properties of the original material of the PCB, for example FR-4. The only essential alteration necessary is to increase the density until the mass of the model is the same as the mass of the PCB, including the electronic components. It is useful to tune the model using a board before processing, adjunsting the length and the structural properties inside the expected range of variation, but the differences are not critical.

It is important to mention that the error may be not in the safety side, so especial attention must be paid by the design engineers.

Example

A rectangular 114.5 x 24 x 1.6 mm board made of FR-4 was modelled using the previous method. The original density of the FR-4, 1870 kg / m^3, was increased up to 3487 kg / m^3 in order to simulate the mass of the electronic components.

A FR-4 board was used for tuning the model. More details about the whole process can be found in APPENDIX: EXPERIMENTAL TESTING - BOARD MODEL. The results of a FEM conducted on the model are shown in Table 3-1. The error is 9.79 %, which is a reasonable approximation.

First resonant frequency (Hz)		
Method	Board	Final PCB
Experimental	552.5	370
FEM	554.74	406.22

Table 3-1.- Comparison of theoretical and experimental results.

3.4 RANDOM VIBRATION

Although sinousidal vibration have been used through all the former sections, it is important to devote some time to the study of random vibration, since it appears in reality and is frequently used in qualification tests. A brief overview will be conducted here, and more information can be found in vibration text books.

The main characteristic of random vibration is that it is nonperiodic, with all frequencies within a bandwith being present all of the time. It is therefore possible to make a probabilistic analysis of the process, but not to predict a precise magnitude in a given instant.

Random vibration is then studied in the frequency domain, through the mathematical function kwnon as Fourier transform. The power spectral density (PSD), which is the main parameter of the excitation, is calculated this way, although during this document an easier and less formal definition will be used:

$$P_{in} = \lim_{\Delta f \to o} \frac{G_{in}^{\ 2}}{\Delta f} \qquad (3\text{-}25)$$

where G_{in} is the root mean square (RMS) of the acceleration input and Δf the bandwith of the frequency range. White noise curves present a constant level for a given frequency span, but it is also possible to work with shaped curves. Since random vibration is not deterministic, the value given in the curve is not the actual acceleration, but a statistic value. In fact the acceleration presents a Gaussian distribution, with the standard deviation σ being equal to the RMS of the acceleration. This probabilistic character of the random vibration will be used in the evaluation of fatigue life (see RANDOM FATIGUE LIFE).

When a PCB is subject to random vibration, experimental data [1] shows that the fundamental resonant frequency experiments large amplitude, while the higher resonance frequencies come through much lower amplitudes. This means that a PCB can be analysed as a single degree of freedom system, whose response to random excitation is to vibrate at its resonant frequency but with varying displacement amplitude. The response can be obtained from the transmissibility T and the input PSD P_{in} as

$$P_{out} = T^2 \, P_{in} \qquad (3\text{-}26)$$

When the random vibration curve is nearly flat in the area of the resonance, the RMS response of the mass can be obtained by considering the area under the PSD-frequency curve of the response

$$G_{out}^{\ 2} = \text{area} = \int_{f_1}^{f_2} P_{out} \, df \qquad (3\text{-}27)$$

For a slightly damped system this can be approximated as

$$G_{out} = \sqrt{\frac{\pi}{2} P_{in} \; f_n \; T} \qquad (3\text{-}28)$$

where f_n is the resonant frequency and P_{in} and T the input PSD and transmissibility at resonance, respectively.

3.5 ESTIMATION OF PARAMETERS

Once the model is choosen, it is necessary to obtain the values of the different parameters which will be used during the analysis. As a general remark, it is always desirable to rely on experimental data. If it is not available it is possible to use figures provided by the manufacturers of the elements involved. Even in that case it usually happens that the information is not a precise value, but an expected range, so experimental validation will be again interesting.

There will be cases, however, in which this is impossible to accomplish, for example when the data is necessary for the design process, before there is any sample for the testing to be conducted. It is therefore important to be able to accurately estimate those paramenters in a preferably simple way.

Material properties

Diverse material and structural properties will be requiered during the different analysis executed, such as density, elastic modulus or Poisson ratio. Most of them will refer to the PCB, since it will constitute the main part of the dynamic study, but information about other elements such as electrotic components or solder material can also be necessary.

The experimental obtaining of this information, which in most cases is not trivial, can be especially difficult when the experimental samples are as small and complex as the usually found in the field of electronic equipment. It is useful to use information from providers or handbooks as a starting point for the testing, or for the final values when it proves to be unsuccesful. It is suggested to build a database with all the data suitable of being necessary.

Natural frequency of the PCB

The first natural frequencies (usually just the first one) of the printed board are usually the most important parameter of the system. They determine the range in which the excitation of the machine will probably end up in a failure. It is therefore essential to obtain an easy but accurate way to estimate them, although as usually experimental data will be the ideal for both initial estimations and verification of theoretical analysis.

As it has been explained in BOARD MODEL, a plate model of the PCB usually provides accurate results in the prediction of the first natural frequency. It is an extremely simple and well studied problem, with several equations which can be found in several mechanical text books. It is more interesting however to conduct an also simple FEM analysis, since it will provide not only the complete set of eigenfrequencies of the board (which will be accurate up to the at least the second mode, according to APPENDIX: EXPERIMENTAL TESTING - BOARD MODEL), but the different shapes related to them, information that may also be useful.

As a reference it can be said that most of the PCBs traditionally found in industry have a resonant frequency between 200 and 300 Hz [1].

Transmissibility of the PCB

Once the fundamental natural frequency of a PCB is kwnon, a critical part of the vibration design of a PCB is the analysis of the dynamic loads developed in the board while resonance. If possible, test data are doubtless the best source of information on the transmissibility characteristic of a PCB, which will depend essentially on the material, the size and the boundary conditions of the PCB. Therefore, it would be interesting to have a reference record from which predict the response characteristics of the boards which are still not manufactured.

A FEM analysis can provide also with data about the transmissimilibity, but its accuracy is highly reduced in resonance conditions. It is a great backdraw, since the knowledge of the transmissibility is especially interesting during the resonance.

Sometimes, however, it is necessary to estimate the tranmissibility without any test data. There are a number of factors that should be considered, all related to the damping characteristics of the board, which determine the amount of energy lost during the vibration, the main reasons being hysteresis due to internal strains and friction with mounting surfaces or ribs. These energy loses depend on the deflections, being greatest when the deflections are greatest. Since deflections are small when the frequency of vibration is high and vice versa, it is logical to anticipate a relation between the transmissibility and the resonant frequency of the board.

According to [1], test data show that in fact transmissibility of a PCB during resonance conditions can generally be related to the square root of its natural frequency, multiplied by a factor usually ranging from 0.5 to 2, which depends upon many factors, such as the following:

- Terms such as “high” or “low” resonant frequency, although only relative, are important. Usually the term low applies to frequencies below 100 Hz and high to frequencies above 400 Hz. A low natural frequency means high displacements and strains, so damping due to enegy losses is high and the transmissibility decreases. In high frequency the displacements are small, so the transmissibility is bigger.

- The input force is also important. A lower input force means low displacements and high transmissibility. On the contrary, a higher input force means high displacements and low transmissibility.

- Small elements will produce no effect over the PCB, while elements covering a bigger span of the board will resulte in larger relative deflections and greater damping. This effect is, however, very small.

- Some of the usual elements of a PCB will have effect on the transimissibily. Riveted and bolted ribs and circuit supports provide a high-pressure interface that will dissipate energy. The same can be said about connectors, heat-sink strips and the extra layers of multiple-layer circuit boards. As usual, the effect will be much less important at high frequency. It is important to state that some elements, such as welded, cast or cemented ribs are, however, too stiff to really dissipate energy, although they will raise the resonant frequency more.

Experience obtained from an experimental data record should provide with the experience to know how to properly apply the previous indications and make accurate estimations.

Example

A small PCB with small electronic components and a resonant frequency of about 400 Hz can have a transmissibility as high as 2 times the square root of the natural frequency for an input force below a 2 G peak, while a big PCB with large electronic components and stiffening ribs, with a resonant frequency of about 100 Hz, can have a transmissibility of about 0.5 times the square root of the natural frequency when the input force is above a 10 G peak.

Excitation

While not a part of the PCB, another essential parameter of the model is the excitation. It is necessary to obtain an accurate chacterization, which is very difficult in a theoretical way, since the different vibrations produced in the machine or tool can be amplified or multiplied during its tranmission process, with new frequencies appearing. Experimental analysis is not trivial either. The machine is not yet built at the design stage and even with a fully operating one several difficulties appear, due to technical reasons and the different possible sources for the critical excitation: the normal service of the tool, an unbalanced revolving component which is normally not expected or even by special external conditions (as it happens with squeaking joints in a nutrunner).

It is important to create a reference record with the data obtained from different kind of already existing machines or tools, even if they do not present problems due to vibration, storing the type of machine, the type of excitation and its relevant data. Then it would be possible to predict the conditions in the tool to design in a much easier way.

When studying the excitations on a machine, the first point is to quantify and classify them as sinusoidal or random vibration. Then every excitation must the characterized with range of frenquencies, acceleration level and regime.

The frequency of the excitation is possibly its essential parameter. Its main application is to compare it with the resonance frenquency of the PCB. As it have been stated before, if both are different the response will be greatly attenuated, so the system will be secure unless the external conditions are extremely harsh. On the contrary, if the PCB has a natural frequency on the range of the excitation, a little vibration input can produce critical damage to the system. In this case a qualitative analysis will not be enough, and the acceleration level will become necessary, since it determines the ultimate response of the system.

The regime of the vibration will be necessary when estimating the life of the machine, due to the accumulative character of the fatigue process. A small but very frequent excitation can produce a faster than a higher excitation which only applies in rare and specific situations. More information can be found on FATIGUE.

Single degree of freedom model

In the cases when the single degree of freedom system is used to model a real system, it will be necessary to estimate its paremeters, that is, mass, stiffness and damping.

The equivalent mass and stiffness would depend on the way the system behaves in its first vibration mode. It depends then not only on its geometry, but also on the boundary limits: the values for a beam would be different if it is clamped in both sides or only in one. It is possible to obtain the equivalent mass-spring model of the defformation of a board with expressions that can be found in any mechanical engineering book. However, those parameters are only used for estimating the natural frequency of the board, so it is usually much easier to just experimentally measure it. This will usually be the method to follow when it is possible.

The damping is obtained also experimentally, from the response curve of the system, as

$$\xi = \frac{\Delta f}{2\,f} \qquad (3\text{-}29)$$

where f is the resonant frequency of the first mode, and Δf the frequency interval between the two points in which the transmissibility is half of the resonance one. This technique is only useful with slightly damped systems, but it is the case of PCBs. Theoretically it works with measurements taken in any point of the board, since it is a global property.

It is important to make sure that the frequency resolution is fine enough. A bad resolution could suggest a resonance peak value much lower than the actual one, so the interval Δf will be bigger than the real one.