8. DAMPING

Appart from stiffening the structure to increase the natural frequencies of the PCB above the excitation frequency, the other fundamental method to protect electronic equipment is to reduce the resonant amplitudes by damping. It works especially well when applied to a PCB with a low resonant frequency, below about 100 Hz. The choice of the elastic and damping properties of the isolation is done according to an optimisation analytical procedure, which varies with the existing casuistry. During the process the different parts involved in the vibration will be modelled as single degree of freedom systems, dynamically characterized by their natural frequency Ω and loss factor ζ. The main differences will stem from the number of systems considered, and in which will be applied the isolation system. For example, in traditional design the PCB will be included in an electronic box, with two clearly different systems (as in MULTYBODY SYSTEMS), but it is also possible that the the PCB is placed in direct contact with the source of vibration, so only one system can be isolated and studied.

8.1 ONE DEGREE OF FREEDOM

If only one system is considered, the model will be the one detailed in SINGLE DEGREE OF FREEDOM MODEL and shown in Fig. 8.1, with two degrees of freedom, which are y(t), the absolute motion of the base, and x_1(t), the motion of the mass, in this case the PCB.

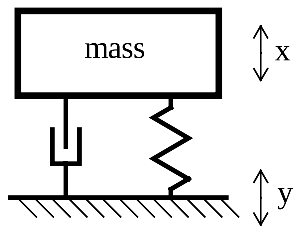

Fig. 8.1 Single of freedom system

It has already been stated that the transmissibility and the frequency in which maximum relative deflection takes place are respectively

$$T_1(\omega) = \sqrt{\frac{\Omega_1^{\ 4} + 4\omega^2\,\Omega_1^{\ 2}\,\xi_1^{\ 2}}{\left(\Omega_1^{\ 2} - \omega^2\right)^2 + 4\omega^2\,\Omega_1^{\ 2}\,\xi_1^{\ 2}}} \qquad (8\text{-}1)$$

$$\omega^* = \Omega_1\sqrt{1 - 2\,\xi_1^{\ 2}} \qquad (8\text{-}2)$$

Introducing the latter in the former it is possible to obtain the expression for the maximum deflection

$$Z_1(\omega)_{max} = Z_1(\omega^*) = \frac{\ddot{Y}}{2\,\Omega_1^{\ 2}\,\xi_1\sqrt{1-\xi_1^{\ 2}}} \qquad (8\text{-}3)$$

This expression can be minimized by increasing the resonant frequency or by adjusting the loss factor to its optimum value, which in this case is $\xi_1 = \frac{1}{\sqrt{2}}$. This will reduce the deformation in the PCB, which is the essential aim of the design process. As a guide, the equation for maximum defformation given in RESTRICTION IN MAXIMUM DISPLACEMENT can be used as an upper limit.

It is important to mention that, according to the simple theory used, those measures will not have the same effect on the transmissibility. In fact, the maximum value of the transmissibility will remain the same, and the main effect of increasing the resonant frequency will be to displace the transmissibility curve to the high frequency range. The reason is that the maximum value of (8–1) is independant of the resonant frequency of the system, although experimental testing shows that in the real world an increase in resonant frequency produces an increase in transmissibility.

8.2 TWO SYSTEMS

This is the approach traditionally followed for electronic structures. Usually the systems are the enclosure (electronic box) as the primary system and the PCB inside it is the secondary system. It would be possible also to model any other case when there is an intermediary element between the excitation and the PCB. In any case the model is as seen in Fig. 8.2, and besides the absolute defelctions $X_1(t)$ and $X_2(t)$, the relative deflections are $Z_1=X_1-Y$ and $Z_2=X_2-Y$. As it is usual in electronic structures it is assumed that the mass of the secondary system (the PCB) is small enough to not affect the dynamics of the primary system. The system can then be studied as two single degree of freedom systems, each of them independant of the other, except for the fact that the output for the primary system is the input of the secondary system.

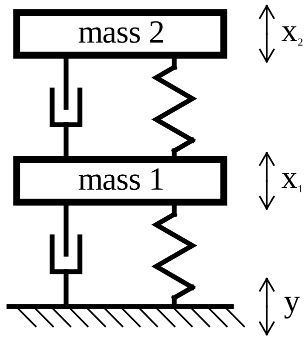

Fig. 8.2.- Two degrees of freedom system.

The traditional isolator design is based on the fact that the attenuation of vibration starts from the frequency $\sqrt{2}\,\Omega$. The idea is then to decrease the resonant frequency of the primary system, while achieving the highest resonant frequency possible for the secondary system. This way the primary system will act as an isolator, reducing the excitation in the secondary system. At the same time, the distance between both resonance frequencies will assure that there is no coupled amplification.

The only restriction is due to the maximum displacement of the primary system, which still is

$$Z_1(\omega)_{max} = \frac{\ddot{Y}}{2\,\Omega_1^{\,2}\,\xi_1\sqrt{1-\xi_1^{\,2}}} \tag{8-4}$$

It is clear that when the resonant frequency tends to zero the denominator of the expression does the same. In the real world this would not automatically imply that the displacement tends to infinite, as the numerator also goes to zero, since $\ddot{Y} = \omega^2\,Y$. The usual assumption of constant $\ddot{Y}$ is then only realistic when frequency is not near zero.

Still, a very low frequency will produce a very flexible system, with great displacements. It is necessary to decide a limit in the form

$$|Z_1| \leq \Delta \tag{8-5}$$

This condition should be based on experience, regulation reliability requierements and spatial restrictions on the design. It can be imposed to the expression in (8-4) and translated into a lower limit for the frequency

$$\Omega_1 \geq \sqrt{\frac{\ddot{Y}}{2\,\Delta\,\xi_1\sqrt{1-\xi_1^{\,2}}}} \tag{8-6}$$

which is again minimized when $\xi_1 = \frac{1}{\sqrt{2}}$. These values give the maximum attenuation over the widest possible frequency range.

A new approach suggested in [11] pursues a reduction of the dynamic response of the internal elements, that is, the PCB, with the electronic box utilized as the first level of vibration isolation (mechanical low-pass filter). This new method minimizes the vibration transmitted to the critical internal components by imposing restraints on the peak deflections of the electronic device. Mathematically this is expresed as

$$\min \left|\ddot{X}_2(\omega)\right|_{max} \qquad \text{or} \qquad \min \left|Z_2(\omega)\right|_{max}$$

Both magnitudes can be developed as seen in MULTYBODY SYSTEMS as

$$\left|\ddot{X}_2(\omega)\right|_{max} = \left|\ddot{X}_2(\Omega_2)\right| = \left|T_1(\Omega_2)\right|\,\left|T_2(\Omega_2)\right|\,\ddot{Y}_0 \tag{8-7}$$

$$\left|Z_2(\omega)\right|_{max} = \left|Z_2(\Omega_2)\right| = \frac{1}{\Omega_2}\left|T_1(\Omega_2)\right|\,\left|T_2^{rel}(\Omega_2)\right|\,\ddot{Y}_0 \tag{8-8}$$

where

$$\left|T_1(\omega)\right| = \sqrt{\frac{\Omega_1^{\,4} + 4\omega^2\,\Omega_1^{\,2}\,\xi_1^{\,2}}{\left(\Omega_1^{\,2}-\omega^2\right)^2 + 4\omega^2\,\Omega_1^{\,2}\,\xi_1^{\,2}}} \tag{8-9}$$

$$\left|T_2(\omega)\right| = \sqrt{\frac{\Omega_2^{\,4} + 4\omega^2\,\Omega_2^{\,2}\,\xi_2^{\,2}}{\left(\Omega_2^{\,2}-\omega^2\right)^2 + 4\omega^2\,\Omega_2^{\,2}\,\xi_2^{\,2}}} \tag{8-10}$$

$$\left|T_2^{rel}(\omega)\right| = \frac{\omega^2}{\sqrt{\left(\Omega_2^{\,2}-\omega^2\right)^2 + 4\omega^2\,\Omega_2^{\,2}\,\xi_2^{\,2}}} \tag{8-11}$$

It has been implicitly assumed that $\ddot{Y}$ is constant. Again, this is only realistic when the frequency is far from zero, which is the usual situation. In other cases, or when the excitation is variable on frequency, a more careful analysis will be necessary.

Since the loss factor of the PCB, ξ_2, is expected to be very small, the equations for $\left|\ddot{X}_2(\omega)\right|_{max}$ and $\left|Z_2(\omega)\right|_{max}$ can be expressed as

$$\left|\ddot{X}_2(\omega)\right|_{max} = \left|\ddot{X}_2(\Omega_2)\right| = \frac{1}{2\,\xi_1}\left|T_1(\Omega_2)\right|\ddot{Y}_0 = \frac{1}{2\,\xi_1}\sqrt{\frac{\Omega_1^{\,4}+4\Omega_2^{\,2}\,\Omega_1^{\,2}\,\xi_1^{\,2}}{\left(\Omega_1^{\,2}-\Omega_2^{\,2}\right)^2+4\Omega_2^{\,2}\,\Omega_1^{\,2}\,\xi_1^{\,2}}}\;\ddot{Y}_0$$

(8-12)

and

$$\left|Z_2(\omega)\right|_{max} = \left|Z_2(\Omega_2)\right| = \frac{1}{2\,\Omega_2^{\,2}\,\xi_1}\left|T_1(\Omega_2)\right|\ddot{Y}_0 = \frac{1}{2\,\Omega_2^{\,2}\,\xi_1}\sqrt{\frac{\Omega_1^{\,4}+4\Omega_2^{\,2}\,\Omega_1^{\,2}\,\xi_1^{\,2}}{\left(\Omega_1^{\,2}-\Omega_2^{\,2}\right)^2+4\Omega_2^{\,2}\,\Omega_1^{\,2}\,\xi_1^{\,2}}}\;\ddot{Y}_0$$

(8-13)

However, since the calculations are surely to be done numerically and the simplification produces no noticeable difference in computational effort, it is suggested to use the complete expressions.

Again, a $\left|Z_1(\omega)\right|_{max} \le \Delta$ restriction must be imposed, that again gives the expression of the resonant frequency

$$\Omega_1 = \sqrt{\frac{\ddot{Y}_0}{2\,\Delta\,\xi_1\sqrt{1-\xi_1^{\,2}}}} \qquad (8\text{-}14)$$

Introducing this expression in (8-12) and (8-13) eliminates Ω_1 and create two expressions which can be numerically minimized, finding the optimal value of the loss factor ξ_1.

Example

The optimum damping system for a PCB of Ω_2 = 350 Hz and ξ_2 = 0.015 will be calculated. The excitation is sinusoidal vibration of $\ddot{Y}_0 = 20\text{ g}$ in frequency range 20 – 600 Hz. The PCB will be placed in an electronic box, whose dynamic properties must be optimized.

The results provided by both the classic and new approach are

$\Omega_{1,classic}$ = 70.5 Hz $\quad\quad$ $\xi_{1,classic}$ = 0.707

$\Omega_{1,new}$ = 104.5 Hz $\quad\quad$ $\xi_{1,new}$ = 0.234

In Fig. 8.3 the response of both configurations is shown. The maximum deflection obtained with the new approach is a 62.58 % of the achieved with the classic method, which is a noticiable improvement.

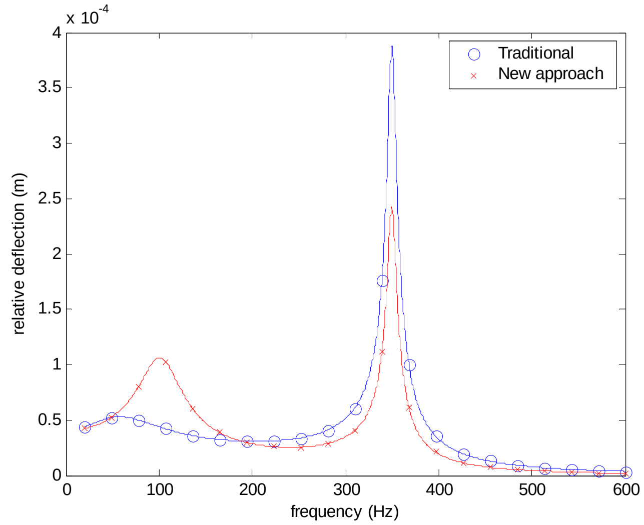

Fig. 8.3.- Damping effect on relative defflection for traditional and new approach

It is important to mention that when the resonant frequency or the damping ratio are specially high the highest response of the secondary system might not appear at Ω_2, but in a previous frequency such as Ω_1. In that case, when using the new approach it is necessary then to calculate the output of the system in all the frequency range for every value of ζ_1, checking for which one the maximum deflection is minimum.

Example

Considering a PCB with Ω_2 = 450 Hz and ξ_2 = 0.025, it is necessary to consider the response of the system in all the frequency range. This way the deflection during the resonance of the secondary system is a little higher, but the maximum deflection is minimum, as can be seen in Fig 8.4.

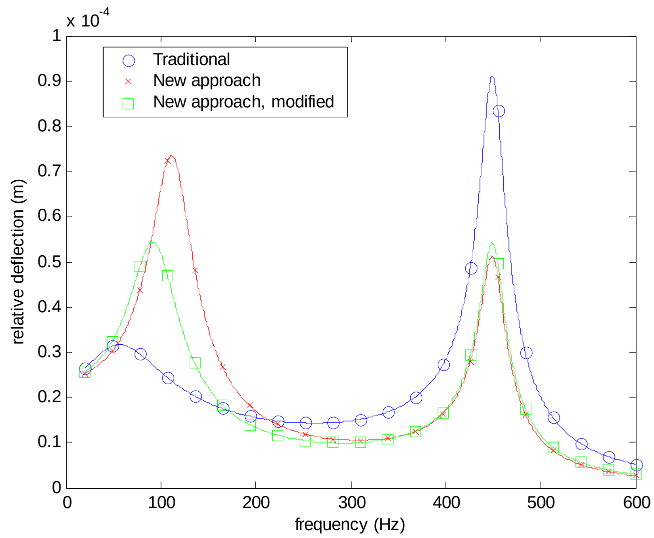

Fig. 8.4.- Damping effect on relative defflection for traditional and new approach

The results provided by each method are the following

$\Omega_{1,classic}$ = 70.5 Hz
$\Omega_{1,new}$ = 114.5 Hz
$\Omega_{1,new\ modified}$ = 96.9 Hz

$\xi_{1,classic}$ = 0.707
$\xi_{1,new}$ = 0.193
$\xi_{1,new\ modified}$ = 0.275

8.3 SUMMARY

The following is a table trying to systematize the three different cases. In the case of the two systems analysis, it has been assumed that the primary is the electronic enclosure and the secondary the PCB.

Method	One system	Two systems classic approach	Two systems new approach
System 1	PCB	Electronic box	Electronic box
System 2	-	PCB	PCB
Aim	Maximize Ω_1 subjected to $\lvert Z_1 \rvert \le \Delta$ Optimum value of ξ_1	Minimize Ω_1 subjected to $\lvert Z_1 \rvert \le \Delta$	minimize $\lvert \ddot{X}_2(\omega) \rvert_{max}$ minimize $\lvert Z_2(\omega) \rvert_{max}$ subjected to $\lvert Z_1 \rvert \le \Delta$
Ω_1	Variable to maximize	$\sqrt{\dfrac{\ddot{Y}_0}{2\,\Delta\,\zeta_1\sqrt{1-\xi_1^2}}}$	$\sqrt{\dfrac{\ddot{Y}_0}{2\,\Delta\,\zeta_1\sqrt{1-\xi_1^2}}}$
Ω_2	-	Obtained experimentally or estimated as seen in Natural frequency of the PCB	Obtained experimentally or estimated as seen in Natural frequency of the PCB
ξ_1	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{\sqrt{2}}$	Variable to obtain during the process
ξ_2	-	Calculated by curve fitting with the transmissibility as seen in Single degree of freedom model	Calculated by curve fitting with the transmissibility as seen in Single degree of freedom model
Δ	Obtained from RESTRICTION IN MAXIMUM DISPLACEMENT.	Extrapolated from experience. Otherwise use a modification of the value obtained from RESTRICTION IN MAXIMUM DISPLACEMENT, since the condition was intended to be applied on the PCB	Extrapolated from experience. Otherwise use a modification of the value obtained from RESTRICTION IN MAXIMUM DISPLACEMENT, since the condition was intended to be applied on the PCB

Table 8-1.- Summary for the different cases of damping design.

8.4 DAMPING FOR RANDOM VIBRATION

The former procedure can be easily adapted to random vibration enviroments, as explained in [12]. The excitation will be characterized by the power spectral density of the acceleration, $P_{\ddot{y}}$. In that case the PSD of the response of the primary and secondary systems can be expressed as

$$P_{\ddot{x}_1}(\omega) = \left|T_1^{abs}(i\,\omega)\right|^2 P_{\ddot{y}}(\omega) \qquad P_{z_1}(\omega) = \frac{1}{\omega^4}\left|T_1^{rel}(i\,\omega)\right|^2 P_{\ddot{y}}(\omega) \qquad (8\text{-}15{,}16)$$

$$P_{\ddot{x}_2}(\omega) = \left|T_2^{abs}(i\,\omega)\right|^2 P_{\ddot{x}_1}(\omega) \qquad P_{z_2}(\omega) = \frac{1}{\omega^4}\left|T_2^{abs}(i\,\omega)\right|^2 P_{\ddot{x}_1}(\omega) \qquad (8\text{-}17{,}18)$$

The root mean square values of the variables are

$$\sigma_{\ddot{x}_1} = \sqrt{\frac{1}{2\,\pi}\int_{-\infty}^{\infty} P_{\ddot{x}_1}(\omega)\,d\,\omega} \qquad \sigma_{z_1} = \sqrt{\frac{1}{2\,\pi}\int_{-\infty}^{\infty} P_{z_1}(\omega)\,d\,\omega} \qquad (8\text{-}19{,}20)$$

$$\sigma_{\ddot{x}_2} = \sqrt{\frac{1}{2\,\pi}\int_{-\infty}^{\infty} P_{\ddot{x}_2}(\omega)\,d\,\omega} \qquad \sigma_{z_2} = \sqrt{\frac{1}{2\,\pi}\int_{-\infty}^{\infty} P_{z_2}(\omega)\,d\,\omega} \qquad (8\text{-}21{,}22)$$

Now the aim of the process will be

$$\min \sigma_{\ddot{x}_2} \qquad \text{or} \qquad \min \sigma_{z_2}$$

The restriction for the maximum displacement of the primary subsystem will be based on the three-band technique, so it can be expressed as

$$\max(z_2) = 3\,\sigma_{z_2} \leq \Delta \qquad (8\text{-}23)$$

The rest of the process is identical to the used for sinusoidal vibration. There are different approximations to the previous expressions, based on the singe degree of freedom system, as seen in [12]. It is however recomended to use the transmissibility curves obtained from experimental testing, especially if more than one resonant frequency is located in the frequency range of the excitation.

8.5 ELECTRONIC COMPONENTS AS A SYSTEM

A special adaptation of this new approach can be used in the case where there is no electronic box or intermediary element by considering the PCB as the primary system and the electronic component as the secondary system.

In this case the condition

$$\left|Z_1(\omega)\right|_{max} \leq \Delta \qquad (8\text{-}24)$$

should use a value at least as restrictive as the one from RESTRICTION IN MAXIMUM DISPLACEMENT.

The main backdraw is that the resonant frequency of the electronic components is usually very high, in the rage of the thousands of hertzs. Experimental testing in the usual components show no clear resonance frequencies, and the only analytical methods developed are complex and not general [13].

Still, it might be the case of an extraordinarily big component, whose resonant frequency is lower than expected and can be clearly identified. In this case, due to the high value of the resonant frequency of the secondary system, it would be necessary to calculate the output in all the frequency range, as it was explained before.

8.6 ISOLATION CHOICE

The result of the process are the resonant frequency and loss factor desired for the system, that is, the PCB, which will be the guiding parameters in the search of the most suitable damping system from the ones commercially available. It is important to remark that ζ_1 is not the loss factor expected in the suspension, since it also includes the own damping of the primary system before the damping system is settled. The same should be said about the resonant frequency. This can be done experimentally, although some initial analytical estimation would be necessary.

In the model used during the process, the addition of the suspension will be modelled like a new set of spring and damping, but without mass, as seen in Fig. 8.5. It will allow to simply obtain the desired characteristics of the isolation. The spring rate is easy to obtain from a static analysis, since it is known that the resultant of two springs added in serie is

$$k = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} \qquad (8\text{-}25)$$

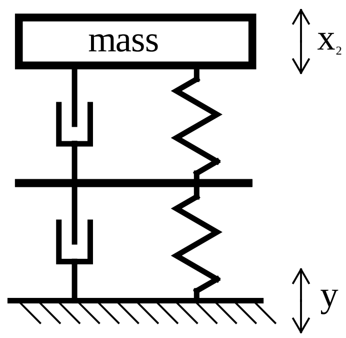

Fig. 8.5.- Mass and isolator model.

where k is the desired spring constant in the global system, equals to $k = m\,\Omega^2$, $k_1 = m\,\Omega_1^{\,2}$ is the spring constant of the PCB and k_2 the spring constant of the isolation. Therefore

$$k_2 = \frac{1}{\frac{1}{k} - \frac{1}{k_1}} = \frac{1}{\frac{1}{m\,\Omega^2} - \frac{1}{m\,\Omega_1^{\,2}}} = \frac{m}{\frac{1}{\Omega^2} - \frac{1}{\Omega_1^{\,2}}} \qquad (8\text{-}26)$$

It is clear that theoretically it is only possible to reduce the resonant frequency of the system.

The values obtained through the methods explained might be difficult to achieve in the practice, due to different restrictions, such as space limitations. Anyway, it is still interesting to have the optimum value as a reference. Also, the equations provided can be used to evaluate the different isolators technically available, in order to select the best of them.