# Chapter 7

# First Variation: The Self-Adapting Test

Results provided by the basic algorithm are not satisfying. The reason, as can be deduced from section 6.3, is the huge range of possible values for the variance of the process. In this chapter we first show (section 7.1) how good the response of the test could be if there were a better approach to the actual value of $\sigma_{proc}$. In section 7.2 the general schema of a self-adapting test is proposed as a solution. Section 7.3 studies the reliability of the inferred value of $\sigma_{proc}$ depending on the sample size $N_2$ and in section 7.4 a criteria is established to adapt the test parameters according to this estimation. Finally, the test is tried and results are shown and commented in section 7.5.

## 7.1 Theoretic minimal reachable delay time values

Figures 6.2 and 6.3 show the average delay time for a medium and a wide test respectively. Both graphics show three lines corresponding to different values the real standard deviation could take. But actually, if the real values of the standard deviation of the process were known, we could choose such a test for which only one line is needed. In the case that $\sigma_{procc} = 194.3683$ only the blue line on figure 6.2 should be taken into account. In the same way, in the case that $\sigma_{proc} = 266.7210$, we could pay attention only to the red line on figure 6.3. Then, delay times would always be much lower than those found in last chapter.

A prove of this is shown on figure 7.1, where the medium test accomplishes the task with much lower delay time values than the wide one when they are applied to a data set whose standard deviation is around 179gr. The reason is that the inferred value of $\sigma_{proc}$ used in the medium test is closer to the real value than the used in the wide test.

Figure 7.2 shows all results together. There, the same lines indicating the average delay time are superposed for the wide and the medium tests and for a new *tight test* (where $\tilde{\sigma}_{proc}$ has the lowest value of the confidence interval). Bold lines indicate the average delay time in the hypothetical case that we could choose the appropriate test depending on the actual value of the standard deviation. In other words, the average delay time if we could always ensure: $\tilde{\sigma}_{proc} = \sigma_{proc}$

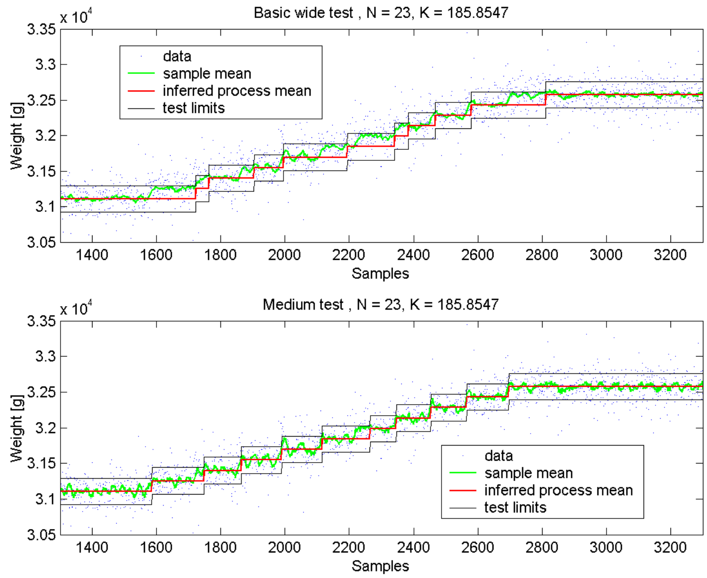


Figure 7.1: Compared results of a wide and a medium test applied to a data set with $\sigma \approx 179$gr.

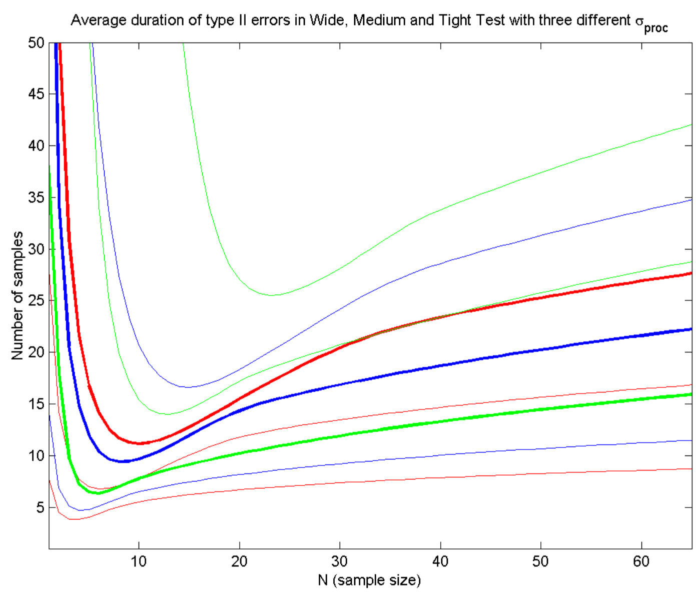


Figure 7.2: Average delay time for different tests and different values of $\sigma_{proc}$. Bold lines represent delay time vs. $N$ with an ideal test with a perfect knowledge of the actual value of $\sigma_{proc}$

If we take a look to the graphic, we see the red line is still the same as with the basic test. But it is different for the other lines. For the case with low real standard deviation (green line), the delay time was expected to be 26 samples using the basic algorithm and now, with an ideal test, this time could be reduced to 6 samples.

In spite of being impossible to know the real value of the standard deviation of a process, an on-line *short term estimation* of this variable could provide a confidence interval which is more narrow than the *long term confidence interval* obtained in chapter 5. A *self-adapting test* would be a good solution as long as we can infer a good approximation to the value of $\sigma_{proc}$.

## 7.2 Description of the self-adapting test

The self-adapting test is similar to the basic one, but the test parameters $N$ and $K$ must be adapted every iteration depending on the current estimation of $\sigma_{proc}$. Figure 7.3 shows a flow diagram describing the new algorithm.

In the diagram, red boxes correspond to new blocks where values of $N$ and $K$ are adapted. In this blocks, the new value of $N$ is first selected attending to the lower limit of the *short term* confidence interval of $\sigma$, and then $K$ is calculated according to the same formula as before, but using the high limit of the confidence interval. Another difference with the basic algorithm are those blocks where it previously said *"estimation"* or *"correction of $\mu$"* and where now it says *"estimation"* or *"correction of $\mu$ and $\sigma$"*. Estimation of $\sigma$ and a rule to adapt $N$ to this value is the most important point in this chapter.

It also must be noted that the estimation of $\sigma$ must be filtered in the same way than the estimation of $\mu$ in order to avoid the high frequency changes produced by statistics as well as the peaks produced by steps in the reading[1].

## 7.3 Maximal confidence interval and minimal sample size for the short-term estimation of $\sigma$ in the self-adaptive test

The estimation of $\sigma_{proc}$ is calculated through the statistic $S$ known as *sample quasi-standard-deviation*, whose expression is

$$S \doteq \sqrt{\frac{1}{N_2 - 1} \sum_{i=1}^{N_2} (x_i - \bar{x})^2}, \tag{7.1}$$

and whose reliability depends on the sample size $N_2$. Since the goal of using this estimation is getting a confidence interval for $\sigma_{proc}$ that is smaller than the one provided in the long term statistical analysis and as reliable as that one, we have to find the limit value $N_{2_{min}}$ necessary for a confidence interval of level 95% to be smaller than the long term one. Let us remember that the size of the long term

[1]See appendix A

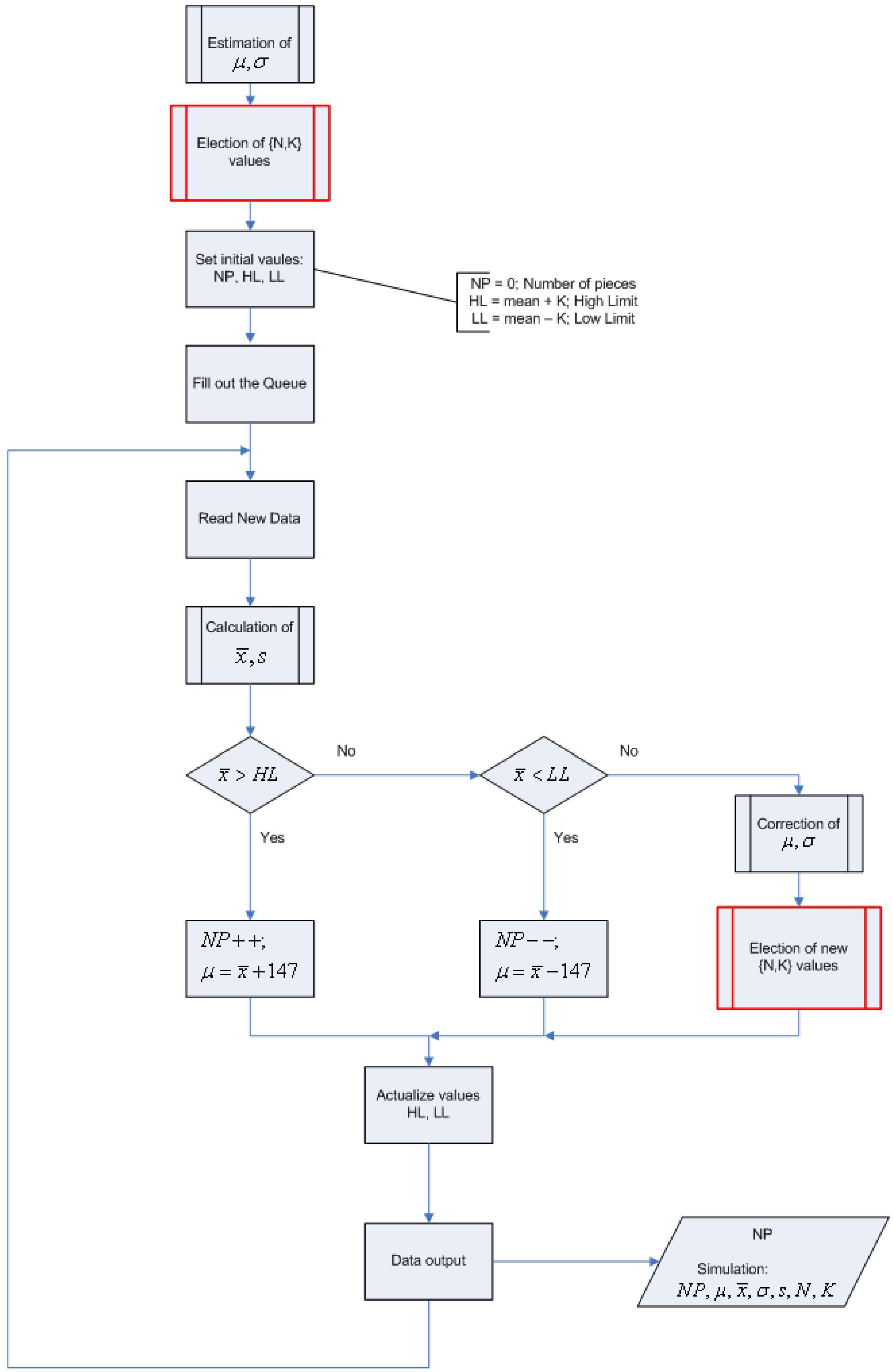


Figure 7.3: Flow diagram for the self-adapting test

confidence interval level 95% is a 74% of the mean value (see section 5.2).

The lowest possible delay time calculated in section 7.1 can only be reached with a perfect estimation of the real value of $\sigma_{proc}$, and, according to the *law of large numbers*, it is only possible by using an infinite number of samples for the estimation, $N_2 = \infty$. Since that is unachievable and since we need a *short term* estimation, we must select any value higher than $N_{2_{min}}$ that leads to short enough delay time values. The method of the adaptive test is considered to be appropriate if there is a normal value of $N_2$ (i.e: not too high to be used in a short term estimation) that results in more accurate estimations of $\sigma_{proc}$ and better average delay times than the obtained up to now.

An easy way to establish a confidence interval for $\sigma_{proc}$ is by using the Tchebysheff's inequality:

$$P(|z - \mu_z| \leq \kappa\sigma_z) \geq 1 - \frac{1}{\kappa^2} \tag{7.2}$$

According to this theorem:

$$\begin{aligned} 1 - \tfrac{1}{\kappa^2} &= 1 - \alpha \\ \kappa &= \tfrac{1}{\sqrt{\alpha}} \end{aligned} \tag{7.3}$$

so a confidence interval for $z$ is defined by:

$$I_{1-\alpha}(z) = E(z) \pm \kappa D(z) = E(z) \pm \frac{D(z)}{\sqrt{\alpha}} \tag{7.4}$$

where: $D^2(z) = Var(z)$.

In our case $z \equiv S$. Now, $E(S)$ and $D(S)$ can be estimated using Fisher's theorem[2]:

$$E(S) = \sigma_{proc} c_2 \sqrt{\frac{N_2}{N_2 - 1}} \tag{7.5}$$

$$D(S) = \sigma_{proc} \sqrt{1 - c_2 \frac{N_2}{N_2 - 1}} \tag{7.6}$$

where:

$$c_2 = \frac{\Gamma(\frac{N_2}{2})}{\Gamma(\frac{N_2-1}{2})\sqrt{\frac{N_2}{2}}} \tag{7.7}$$

[2]This is only an approximation of the real values inasmuch as Fisher's theorem works with normal distributions and ours is not a normal one

With (7.5) and (7.6), expression (7.4) can be written as follows:

$$I_{1-\alpha}(\sigma) = \sigma_{proc}\left(c_2\sqrt{\frac{N_2}{N_2-1}} \pm \sqrt{1-c_2\frac{N_2}{N_2-1}}\frac{1}{\sqrt{\alpha}}\right) \tag{7.8}$$

Thus, $N_{2_{min}}$ is the minimal value of $N_2$ for which:

$$2\sqrt{1-c_2\frac{N_2}{N_2-1}}\frac{1}{\sqrt{\alpha}} \le 0.74 \tag{7.9}$$

Solving equation (7.9) with $\alpha = 0.05$, we get $N_{2_{min}} \geq 74$. This means that a good improvement would be achieved by using values of $N_2$ near to 100 which is actually a huge value.

With regard to this result, it must be noted that the Tchebysheff's inequality provides a coarse interval that is valid for any distribution no matter how weird it is. Thereby, it must be easy to establish a narrow confidence interval for lower values of $N_2$ if we use any information about the actual distribution. This is what we do in the following approach.

A logical and easy solution, since we are already using Fisher's theorem as an approximation, is to consider that $S$ comes from a gaussian population and to use directly the Fisher's theorem to establish an Interval. This can be done inasmuch as:

$$a = \sqrt{\frac{\chi^{2(\alpha)}_{N_2-1}\sigma^2_{proc}}{N_2-1}} \tag{7.10}$$

where:

$$\chi^{2(\alpha)}_{N_2-1} : P(\chi^2_{N_2-1} \le \chi^{2(\alpha)}_{N_2-1}) = \alpha \tag{7.11}$$

is such a value for which:

$$P(S^2 \le a^2) = \alpha \tag{7.12}$$

In this case we are setting only the upper bound of the interval and the lower one will be set symmetric[3]. According to this approach we use $\alpha = 0.025$ and $a = 1.37 \cdot S$, and obtain the following minimal value for $N_2$: $N_{2_{min}} = 15$, which is a much lower value than the previous one. This means that using 27 samples (1 second) to estimate the standard deviation on-line will provide better results than the obtained in the previous chapter. Furthermore, 1 second is a short enough period of time to be used in a *"short term"* estimation and even a higher value could be used if necessary.

As a consequence, if these results are reliable (let us remember that it is only approximation), the self-adapting test can be successfully used for the counting scale. This second value of $N_{2_{min}}$ is used here and *its reliability remains outstanding to be checked depending on the results of the test.*

[3]The computation of a confidence interval relative size depending on $N_2$ and $\alpha$ is implemented in function "ConfidenceInterval.m" for MATLAB. See appendix C.1

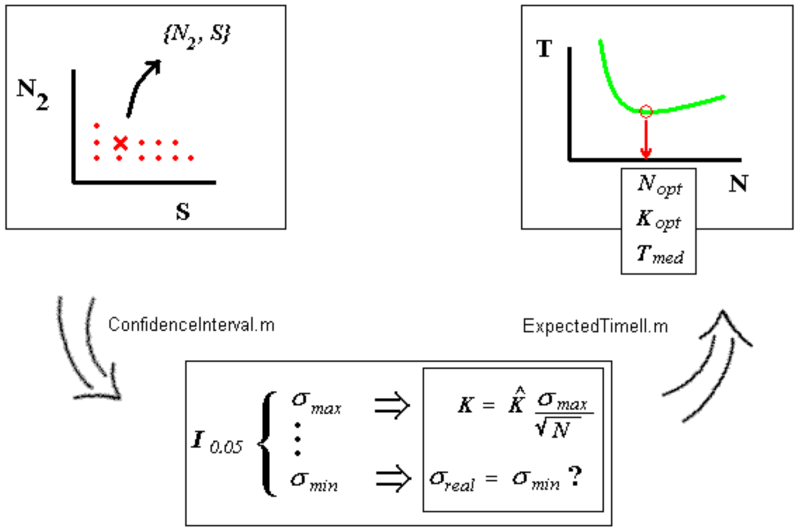


Figure 7.4

$$\boxed{N_{2_{min}} = 15}$$

## 7.4 Optimal values

In the previous section the way to calculate a confidence interval for $\sigma_{proc}$ has been discussed. Now we use these new intervals to repeat the steps in chapter 6 to find the optimal values of $N$ and $K$. While in chapter 6 we had an only solution $\{N_{opt}, K_{opt}, \bar{T}_{delay}(N_{opt}, K_{opt})\}$ for the whole range of $\sigma$ values, in this case a different solution is obtained for each estimation of $\sigma$. Besides, $N_2$ can be arbitrarily selected (provided that it is higher than 15), so there is a new degree of freedom that enables changing the optimal point inside a certain range. So, for this study, optimal values of $N$ and $K$ have been calculated for each pair of values $\{N_2, S\}$.

The way to calculate these values is shown on figure 7.4: For each pair of values $\{N_2, S\}$ the new confidence interval level 95% is calculated according to (7.8)[4]. Then the new diagram showing $T_{delay}$ vs. $N$ is generated as it was in chapter 6[5]. The minimum of the *green line* indicates $N_{opt}$ and $\bar{T}_{delay}$. Then, $K_{opt}$ is calculated using the upper value of the confidence interval as: $K = \hat{K}\hat{\sigma}$.

Same calculations have been repeated[6] for a huge range of $N_2$ and $S$. Mean

[4] Since $\sigma_{proc}$ is unknown in equation 7.8, $\frac{S}{c_2(N_2)}$ is used as its real value

[5] Only the *green line* of the *wide test* is necessary. This is calculated by function expectedtimeII.m when using the upper value of the confidence interval for $\hat{\sigma}$ and the lower value for $\sigma_{real}$. However, lines are expected to be close to each other in this case because the confidence interval for $\sigma$ is reduced for values of $N_2$ higher than 15

[6] The whole calculation procedure represented on figure 7.4 is carried out by function "test_optimo_ConfInterval.m" for MALTAB. This function is called from "OptimalTestSur-

delay time for the optimal values of $N$ and $K$ are shown on figure 7.5. Using these results it would be possible to establish a rule that adapts $N_2$ depending on $S$ in order to keep the same mean delay time for the whole range of $S$. Nevertheless it is not worth it since we can just fix $N_2$ to a value that limits the delay time in the worst cases (very high $S$ values) and results will be better for lower values of $S$.

Let us remember that there is no specification for the delay time of the test, thereby we have to choose our own specification based just on common sense. On the one hand we want to reduce the delay time as much as possible by choosing high values of $N_2$ but on the other hand, too high values of $N_2$ would lead to unfeasible computation load when calculating $S$. In this thesis we consider delay times lower than one second are acceptable. Looking at the graphic, we find this goal is always achieved if $N_2 \geq 26$. This value is not too high and can even be easily doubled. With $N_2 = 52$ the average delay time is around 0.5 seconds and lower than 0.8 for high values of the standard deviation. Higher values of $N_2$ start being too high to be used in a short term estimation since they correspond to sample periods longer than 2 seconds. Despite the fact that sample size values like $N_2 = 60$ could also be used, trials using $N_2 = 52$ provide good results. So that is the choice used in this thesis.

$$\boxed{\begin{aligned} N_2 &= 52 \\ \bar{T}_{delay} &< 0.8 \text{ seconds} \end{aligned}} \tag{7.13}$$

Now using $N_2 = 52$ we get the following values of $N_{opt}$:

Table 7.1: Optimal values for the sample size $N$ and highest values of $S$ for which they are to be applied

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\boldsymbol{N_{opt}}$ | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| $\boldsymbol{S_{max}}$ | 105 | 118 | 130 | 141 | 154 | 167 | 183 | 198 | 211 | 224 | 236 |
| $\boldsymbol{N_{opt}}$ | 18 | 19 | 20 | 21 | - | - | - | - | - | - | - |
| $\boldsymbol{S_{max}}$ | 249 | 262 | 278 | 295 | - | - | - | - | - | - | - |

Values in table 7.1 can be directly implemented in the program on the microprocessor as they are in function `optimalTestValues.m` for MATLAB (see Appendix C.1), so no calculation must be done during the application of the test.

faces.m" for a list of $N_2$ and $S$ values. This has been used to gender figure 7.5. See appendix C.1

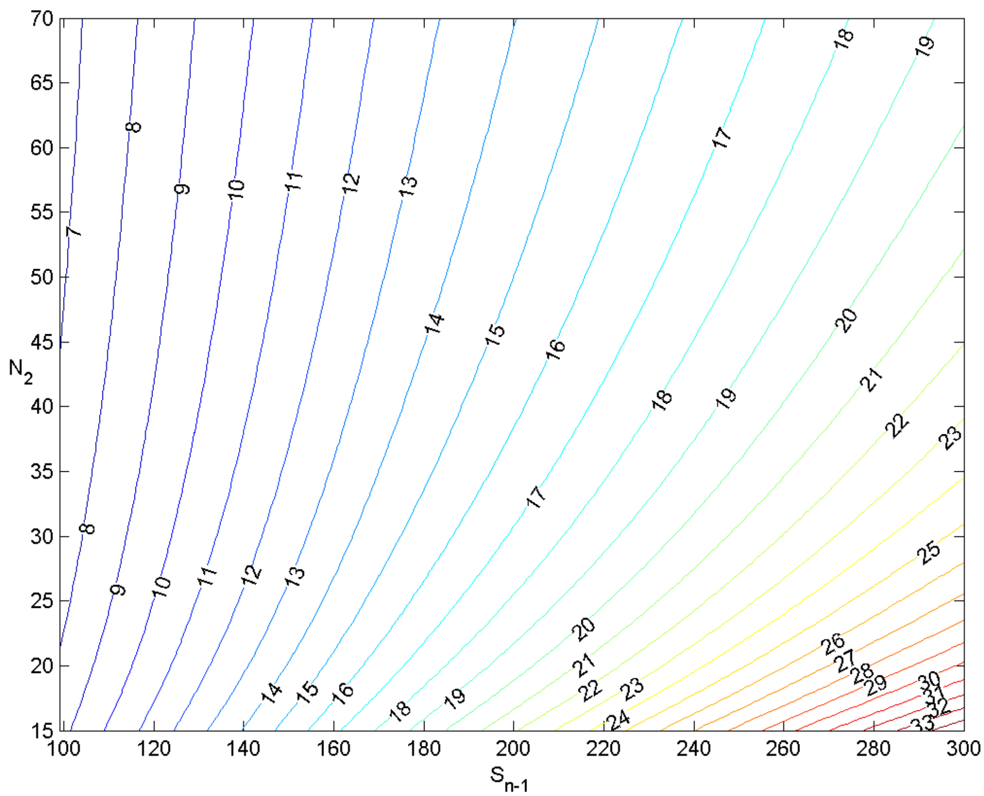


Figure 7.5: Mean delay time (number of samples) for optimal values of $N$ and $K$

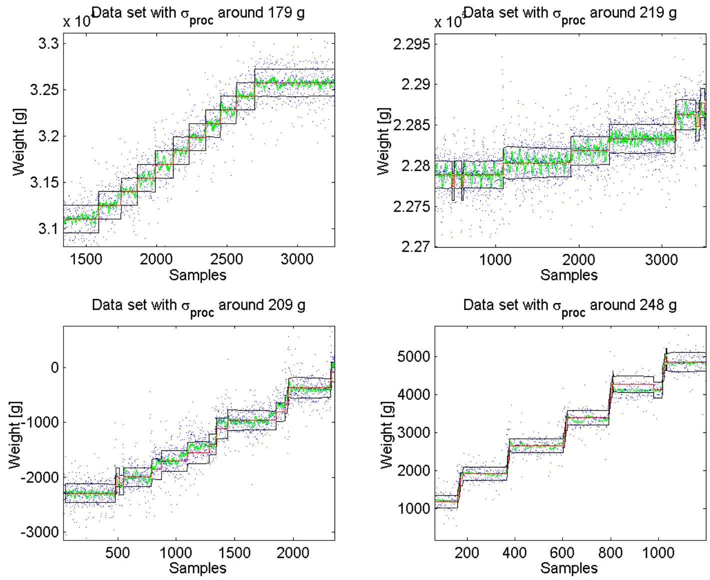


Figure 7.6: Response of the adaptive test for four data sets with different standard deviation

## 7.5 Results and discussion

The self-adapting test has been tried with different data sets including those used in chapter 6 as well as some others not used for the statistical analysis. Figure 7.6 shows the results indicating the standard deviation of the each data set. The chart at the upper left corner corresponds to the same data set in figures 6.5 and 7.1. It can be noted that the response is much better than that obtained with the wide test and even better than with the medium test. Simultaneously, the chart at the upper right corner shows as good results as in figure 6.4 while both of them come from the same data set.

The two charts at the bottom correspond to other irregular data sets where steps are not *one piece high*, but higher. This has been tried to fulfill a wide variety of situations concerning the input of the test. In these cases the response of the test is good in spite of the wide range of standard deviation values we are covering.

In conclusion, the self-adapting test is an appropriate algorithm that performs the count of the units in the scale successfully.

In the next chapter the behavior of the test under vibrations of the ground is studied and a new small variation is introduced to overcome some deficiencies.