Apéndice A

Matrices

En este apéndice se exponen las expresiones explícitas de las matrices y vectores que aparecen en este trabajo.

Las matrices y vectores que aparecen en las ecuaciones de movimientos no lineales discretizadas (2.21) y (2.22) en función de las variables $\theta(\xi)$ vienen dadas por

$$
\begin{aligned}
M_{\theta,in}(\boldsymbol{\theta}) &= \sum_{k=1}^{N} A_{ik} B_{kn} \cos(\theta_i - \theta_n)\,, \\
S_{\theta,in}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= \sum_{k=1}^{N} A_{ik} B_{kn} \dot{\theta}_n \sin(\theta_i - \theta_n)\,, \\
C_{\theta,in}(\boldsymbol{\theta}) &= B_{in} \cos(\theta_i - \theta_n)\,, \\
\boldsymbol{F}_{c,\theta,i}(\boldsymbol{\theta}) &= -\sin(\theta_i - \theta_N)\,, \\
\boldsymbol{F}_{rel,\theta,i}(\boldsymbol{\theta}) &= \sum_{n=1}^{N} -B_{in} \sin(\theta_i - \theta_n)\,, \\
\boldsymbol{F}_{g,p,\theta,i}(\boldsymbol{\theta}) &= (1 - \xi_i) \sin(\theta_i - \theta_G)\,, \\
\boldsymbol{M}^T_{f,\theta,n}(\boldsymbol{\theta}) &= \sum_{i}^{N} A_{in} \sin(\theta_i - \theta_n)/N\,, \\
\boldsymbol{S}^T_{f,\theta,n}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= \sum_{i}^{N} -A_{in} \dot{\theta}_i \cos(\theta_i - \theta_n)/N\,, \\
F_{g,f}(\boldsymbol{\theta}) &= -\sum_{n}^{N} \cos(\theta_n - \theta_G)/N.
\end{aligned}
\tag{A.1}
$$

Mientras, las mismas matrices proyectadas en coordenadas generalizadas dadas en las ecuaciones (2.26) y (2.27) vienen dadas por

$$
\begin{aligned}
M &= \Phi^T M_\theta(\boldsymbol{\theta})\Phi \,, \\
S &= \Phi^T S_\theta(\boldsymbol{\theta})\Phi \,, \\
C &= \Phi^T C_\theta(\boldsymbol{\theta})\Phi \,, \\
K &= \Phi^T K_\theta(\boldsymbol{\theta})\Phi \,, \\
\boldsymbol{F}_c &= \Phi^T \boldsymbol{F}_{c,\theta}(\boldsymbol{\theta}) \,, \\
\boldsymbol{F}_{rel} &= \Phi^T \boldsymbol{F}_{rel,\theta}(\boldsymbol{\theta}) \,, \\
\boldsymbol{F}_{g,p} &= \Phi^T \boldsymbol{F}_{g,p,\theta}(\boldsymbol{\theta}) \,, \\
\boldsymbol{F}_K &= \Phi^T K \boldsymbol{\theta}_{eq} \,, \\
\boldsymbol{M}_f &= \Phi^T \boldsymbol{M}_{f,\theta}(\boldsymbol{\theta}) \,, \\
\boldsymbol{S}_f &= \Phi^T \boldsymbol{S}_{f,\theta}(\boldsymbol{\theta}) \,, \\
F_{g,f} &= F_{g,f,\theta}(\boldsymbol{\theta}) \,,
\end{aligned}
\tag{A.2}
$$

Y las que aparecen en las ecuaciones linealizadas (2.33) y (2.34)

$$
\begin{aligned}
M^{eq} &= \Phi^T M_\theta^{eq}(\boldsymbol{\theta}_{eq})\Phi \,, \\
C^{eq} &= \Phi^T C_\theta^{eq}(\boldsymbol{\theta}_{eq})\Phi^T \,, \\
\boldsymbol{F}_c^{eq} &= \Phi^T \boldsymbol{F}_{c,\theta}^{eq}(\boldsymbol{\theta}_{eq}) \,, \\
K_{F_c}^{eq} &= \Phi^T K_{F_c,\theta}^{eq}(\boldsymbol{\theta}_{eq})\Phi \,, \\
K_g^{eq} &= \Phi^T K_{g,\theta}^{eq}(\boldsymbol{\theta}_{eq})\Phi \,, \\
F_{g,f}^{eq} &= F_{g,f,\theta}(\boldsymbol{\theta}_{eq}) \,, \\
\boldsymbol{F}_{rel}^{eq} &= \Phi^T \boldsymbol{F}_{rel,\theta}^{eq}(\boldsymbol{\theta}_{eq}) \,, \\
\boldsymbol{M}_f^{eq} &= \Phi^T \boldsymbol{M}_f^{eq,T}(\boldsymbol{\theta}_{eq}) \,.
\end{aligned}
\tag{A.3}
$$

y

$$
\begin{aligned}
M^{eq}_{\theta,in}(\boldsymbol{\theta}) &= \sum_{k=1}^{N} A_{ik} B_{kn} \cos(\theta^{eq}_i - \theta^{eq}_n)\,, \\
C^{eq}_{\theta,in}(\boldsymbol{\theta}) &= B_{in} \cos(\theta^{eq}_i - \theta^{eq}_n)\,, \\
\boldsymbol{F}^{eq}_{c,\theta,i}(\boldsymbol{\theta}) &= -\sin(\theta^{eq}_i - \theta^{eq}_N)\,, \\
K^{eq}_{F_c,\theta,in}(\boldsymbol{\theta}) &= \begin{cases} -\cos(\theta^{eq}_i - \theta^{eq}_n) & i = n < N \\ \cos(\theta^{eq}_i - \theta^{eq}_n) & i < n = N \\ 0 & \text{otros} \end{cases} \\
\boldsymbol{K}^{eq}_{g,\theta}(\boldsymbol{\theta}) &= \begin{cases} (1-\xi_i)\cos(\theta^{eq}_i - \theta^{eq}_N) & i = n \\ 0 & i \neq n \end{cases} \\
\boldsymbol{F}_{g,f,i}(\boldsymbol{\theta}) &= \sin(\theta_i - \theta_G)/N\,, \\
\boldsymbol{F}^{eq}_{rel,\theta,i}(\boldsymbol{\theta}) &= \sum_{n=1}^{N} -B_{in} \sin(\theta^{eq}_i - \theta^{eq}_n)\,, \\
\boldsymbol{M}^{eq,T}_{f}(\boldsymbol{\theta}) &= \sum_{i}^{N} A_{in} \sin(\theta^{eq}_i - \theta^{eq}_n)/N\,.
\end{aligned}
\tag{A.4}
$$

Las matrices que aparecen en las ecuaciones de movimiento (3.8) y (3.9) en coordenadas modales vienen dadas por

$$
\begin{aligned}
\hat{M}_m(\boldsymbol{\theta}) &= H^T \begin{pmatrix} I & 0 \\ 0 & M_\theta \end{pmatrix} H \\
\hat{M}^{pf}_m(\boldsymbol{\theta}) &= H^T \begin{pmatrix} 0 \\ \beta \boldsymbol{F}_{rel,\theta} \end{pmatrix} \\
\hat{M}^{fp}_m(\boldsymbol{\theta}) &= \begin{pmatrix} 0 & \beta \boldsymbol{M}_{f,\theta} \end{pmatrix} H \\
\hat{K}_m &= H^T \begin{pmatrix} 0 & I \\ -K_\theta & 0 \end{pmatrix} H \\
\boldsymbol{F}_p(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \mathrm{U}) &= H^T \begin{pmatrix} 0 \\ -K_\theta \boldsymbol{\theta_{eq}} - S_\theta \dot{\boldsymbol{\theta}} - 2\beta \mathrm{U} C_\theta \dot{\boldsymbol{\theta}} - \beta \mathrm{U}^2 \boldsymbol{F}_{c,\theta} - G \boldsymbol{F}_{g,p,\theta} \end{pmatrix} \\
F_f(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \mathrm{U}) &= \Delta P - \frac{1}{2}\beta \mathrm{U}^2 - \beta S^T_{f,\theta} \dot{\boldsymbol{\theta}}^2 - \beta G F_{g,f,\theta}
\end{aligned}
\tag{A.5}
$$