LETTER *Special Section on Information Theory and Its Applications*

A Simulation Probability Density Function Design for TCM Scheme in Impulsive Noise Environment

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SUMMARY We present a design method of the simulation probability density function for a trellis-coded modulation (TCM) in an impulsive noise environment. The upper bound evaluation method for the TCM scheme cannot be applied to the lognormally distributed impulsive noise, since the Chernoff bound cannot be defined. Thus the error probability can only be estimated by a computer simulation. For an evaluation of a low error probability, importance sampling (IS) is an efficient technique. A design method of the simulation probability density function, which plays an important role in IS, is proposed for the noise. The effectivity is shown by a numerical example.
key words: *importance sampling, trellis coding, error event simulation, impulsive noise*

1. Introduction

In an urban area, error probability evaluation in an impulsive noise environment is more important than that in a Gaussian noise environment. When we treat lognormal noise [1], [2] as impulsive noise, it is impossible to evaluate the upper bound of the error performance of the trellis-coded modulation (TCM) scheme since the Chernoff bound cannot be defined. The estimation can only be made by a computer simulation.

The error probability of the TCM scheme is effectively evaluated by the error event simulation method with importance sampling (IS). The IS method is one of the variance reduction simulation techniques, and can reduce the simulation time as compared to the Monte-Carlo (MC) simulation. The reduction of the simulation time depends on the design of the simulation probability density function (p.d.f.) used in IS. An efficient evaluation of the error probability in the impulsive noise environment is expected to include IS. In this paper, we propose a design method of the simulation p.d.f. for impulsive noise.

2. Importance Sampling

Let $\boldsymbol{z}$ be an i.i.d. (identically and independently distributed) random variable with the p.d.f. $f(\boldsymbol{z})$. Consider a probability P given by

$$P = \int_{\Omega} I(\boldsymbol{z}) f(\boldsymbol{z}) \, d\boldsymbol{z}, \tag{1}$$

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where Ω is the whole space and $I(\cdot)$ is the indicator function defined as

$$I(\boldsymbol{z}) = \begin{cases} 1, & \boldsymbol{z} \in D \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where D is the subset of Ω.

In the IS method, the probability is calculated by introducing another p.d.f. $f^*(\cdot)$. Equation (1) is rewritten as

$$P = \int_{\Omega} I(\boldsymbol{z}) \frac{f(\boldsymbol{z})}{f^*(\boldsymbol{z})} f^*(\boldsymbol{z}) \, d\boldsymbol{z}. \tag{3}$$

When $f^*(\cdot) \neq 0$ is satisfied under the condition of $I(\cdot) f(\cdot) \neq 0$, Eqs. (1) and (3) are identical. The p.d.f. $f^*(\cdot)$ is called the simulation probability density function. If the variance of Eq. (3) is less than that of Eq. (1), the simulation time with IS may be reduced in comparison with that with the MC method.

3. Error Event Simulation Method

The error event simulation (EES) method [3] is combined with IS to evaluate the error probability of the TCM scheme. In the EES method, the error probability is evaluated by estimating the probability of each error event.

The convolutional encoder without a parallel path is assumed because we can easily extend the following for the case of a parallel path. Let $\boldsymbol{u} = (\cdots, u_{-1}, u_0, u_1, \cdots) \in \mathcal{U}$ be a correct state sequence, where $\mathcal{U}$ is the set of all correct state sequences. Furthermore, let $\boldsymbol{\varepsilon_u}$ be a set of the error events, which are correctly decoded at time zero and incorrectly decoded at time one. The bit error probability (BER) P_b of the TCM scheme is expressed as

$$P_b = \frac{1}{m} \sum_{\boldsymbol{u} \in \mathcal{U}} \sum_{\boldsymbol{u}' \in \boldsymbol{\varepsilon_u}} n_b(\boldsymbol{u}, \boldsymbol{u}') P_c(\boldsymbol{u}) P(\boldsymbol{u}'|\boldsymbol{u}), \tag{4}$$

where $\boldsymbol{u}'$ is an error event in $\boldsymbol{\varepsilon_u}$, m is the number of information bits, $n_b(\boldsymbol{u}, \boldsymbol{u}')$ is the number of error bits under the condition of the correct state sequence $\boldsymbol{u}$ and the error event $\boldsymbol{u}'$, $P_c(\boldsymbol{u})$ is the probability of the correct state sequence $\boldsymbol{u}$, and $P(\boldsymbol{u}'|\boldsymbol{u})$ is the conditional probability of error event $\boldsymbol{u}'$ under the condition of the correct state sequence $\boldsymbol{u}$. Since the set $\boldsymbol{\varepsilon_u}$ is an infinite

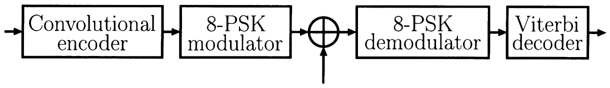

Fig. 1 System model.

set, the BER is approximated by the conditional probabilities for the finite important set $\widehat{\varepsilon}_{\boldsymbol{u}}$ in the practical simulation.

Let $\boldsymbol{f}\left(\boldsymbol{r}|\boldsymbol{s}\left(\boldsymbol{u}\right)\right)$ and $\boldsymbol{f}^*\left(\boldsymbol{r}|\boldsymbol{s}\left(\boldsymbol{u}\right)\right)$ be the original and simulation conditional p.d.f. of the received signal sequence $\boldsymbol{r}$ under the condition that the correct signal sequence is $\boldsymbol{s}\left(\boldsymbol{u}\right)$. After the N_{EES} simulation, the IS estimator of the BER is expressed as

$$\widehat{P}_b^* = \frac{1}{m\cdot N_{EES}}\sum_{\boldsymbol{u}'\in\widehat{\varepsilon}_{\boldsymbol{u}}}\sum_{j=1}^{N_{EES}} n_b\left(\boldsymbol{u},\boldsymbol{u}'\right) \cdot I_{\boldsymbol{u},\boldsymbol{u}'}\left(\boldsymbol{r}^{(j)}\right)\frac{\boldsymbol{f}\left(r_1^{(j)},r_2^{(j)},\cdots|\boldsymbol{s}\left(\boldsymbol{u}\right)\right)}{\boldsymbol{f}^*\left(r_1^{(j)},r_2^{(j)},\cdots|\boldsymbol{s}\left(\boldsymbol{u}\right)\right)}, \quad (5)$$

where $\boldsymbol{r}^{(j)} = \left(r_1^{(j)},r_2^{(j)},\cdots\right)$ is the jth received signal sequence where $r_i^{(j)}$ is the received signal at time i, and $I_{\boldsymbol{u},\boldsymbol{u}'}\left(\cdot\right)$ is the indicator function defined as

$$I_{\boldsymbol{u},\boldsymbol{u}'}\left(\boldsymbol{r}\right) = \begin{cases} 1, & \text{if the error event } \boldsymbol{u}' \text{ is decoded under the condition of the correct state sequence } \boldsymbol{u} \text{ and the received signal sequence } \boldsymbol{r} \\ 0, & \text{otherwise.}\end{cases} \quad (6)$$

4. System Model

Figure 1 shows the system model. The information sequence is encoded and is modulated to the 8-PSK (phase shift keying) signal sequence. The transmitted signal is disturbed by an additive impulsive noise. As a noise over the channel, only the additive impulsive noise is considered [2]. The received signal is demodulated and is decoded by the Viterbi decoder. The squared Euclidean distance is used as the metric in the Viterbi decoder because the branch metric cannot be defined.

5. Noise Model

In this paper, we treat only impulsive noise, namely, we ignore background noise. For the p.d.f. of the amplitude of the impulsive noise, the lognormal distribution is used [1], [2]. As the random variable of the p.d.f. of the amplitude of the noise, the normalized value γ defined as $\gamma = \Gamma/\left(AT\right)$ is used, where Γ is the area of the impulsive noise per symbol interval T and A is the amplitude of the 8-PSK signal. The p.d.f. $W\left(\gamma\right)$ of the lognormal noise is described as

$$W\left(\gamma\right) = \begin{cases}\frac{1}{\sqrt{2\pi}\sigma\gamma}\exp\left(-\frac{\left(\ln\gamma-\mu\right)^2}{2\sigma^2}\right), & \gamma\geq 0 \\ 0, & \gamma < 0,\end{cases} \quad (7)$$

where σ^2 and μ are the variance and mean value of $\ln\gamma$, respectively. The lognormal distribution is presented by the skewness parameter B defined as

$$B = 20\log_{10}\frac{\sqrt{\overline{\gamma^2}}}{\overline{\gamma}}. \quad (8)$$

By using B, the parameters can be rewritten as

$$\sigma^2 = \frac{B\ln 10}{10} \quad (9)$$

and

$$\mu = \frac{1}{2}\ln\overline{\gamma^2} - \frac{B\ln 10}{10}. \quad (10)$$

The phase of the impulsive noise is uniformly distributed and the impulsive noise occurs with probability $\nu T\left(\ll 1\right)$ per symbol interval [1], [2].

The expression for the the signal-to-noise ratio S/N used in the abscissas of the graphs deserves a brief explanation. If we assume that the equivalent noise impulses at the receiver input have times of arrival which are Poisson-distributed and independent complex areas, the power spectrum of the real input noise process will be white with spectral density

$$N = \frac{1}{2}\overline{\Gamma^2}\nu. \quad (11)$$

Therefore, S/N in the information bandwidth $1/T$ is

$$S/N = \frac{1}{\nu T\overline{\gamma^2}}. \quad (12)$$

6. Proposed Simulation Probability Density Function

Let $x+\jmath y$ be an additive complex impulsive noise on a signal space in each time instant. The noise p.d.f. over the channel is rewritten as

$$f_n\left(x,y\right) = \left(1-\nu T\right)\delta\left(x,y\right) + \nu T f_{ln}\left(x,y\right), \quad (13)$$

where $f_{ln}\left(\cdot,\cdot\right)$ is the noise p.d.f. As noted above, the amplitude and phase of the noise are lognormally and uniformly distributed, respectively. Since the nth power of the delta function $\delta\left(\cdot,\cdot\right)$ is not defined except for $n=1$, the twisted distribution [4] cannot be applied to the design of the simulation p.d.f. Hence, instead of the twisted distribution, the simulation p.d.f. is separately designed as

$$f_n^*\left(x,y\right) = \left(1-\nu^* T\right)\delta\left(x,y\right) + \nu^* T f_{ln}^*\left(x,y\right), \quad (14)$$

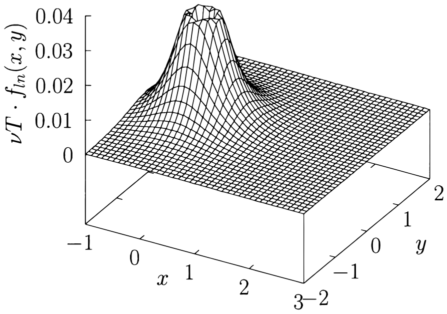

Fig. 2 Original probability density function $\nu T \cdot f_{ln}(\cdot,\cdot)$. Correct signal is at $(0,0)$. The parameters are $B=2$, $\nu T=0.01$, and $S/N=20$ [dB].

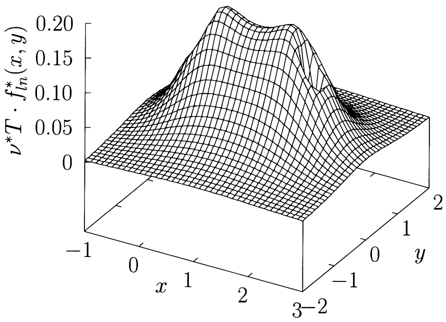

Fig. 3 Simulation probability density function $\nu^* T \cdot f^*_{ln}(\cdot,\cdot)$. Correct and target signals are at $(0,0)$ and $(2,0)$, respectively. The parameters are $B=2$, $\nu T=0.01$, $\nu^* T=0.2$, and $S/N=20$ [dB].

where $f^*_{ln}(\cdot,\cdot)$ is the simulation p.d.f. corresponding to $f_{ln}(\cdot,\cdot)$ and $\nu^* T$ is a parameter used to speed up the simulation ($\nu T \le \nu^* T < 1$).

The simulation p.d.f. $f^*_{ln}(\cdot)$ designed by the twisted distribution [4], [5] is equal to $f_{ln}(\cdot)$ since the simulation p.d.f. cannot be derived under other conditions. The simulation p.d.f. related to the Bhattacharyya bound [6], which is asymptotically optimal for the Viretbi metric, is used as $f^*_{ln}(\cdot)$. Let $f^*(\cdot|\cdot)$ be the simulation p.d.f. related to the Bhattacharyya bound. The conditional simulation p.d.f. $f^*(\cdot|\cdot)$ is written as

$$f^*(r|c) = K\sqrt{f(r|c)\,f(r|e)}, \tag{15}$$

where K is the constant required to make $\int_\Omega f^*(r|\cdot)\,dr = 1$, $f(\cdot|\cdot)$ is the conditional p.d.f. of the noise, and c, e, and r are the correct signal, the target error signal, and the transmitted signal, respectively. Figures 2 and 3 show examples of the original and simulation p.d.f. of the noise, respectively.

The speedup parameter $\nu^* T$ should be selected to minimize the simulation time under certain variances of the estimator. The variance under fixed simulation runs becomes large when $\nu^* T \to \nu T$ and $\nu^* T \to 1$. The reason for the former is that the frequency of the target error event decoded by IS is too small. The case of the latter is that the simulation p.d.f. is very different from

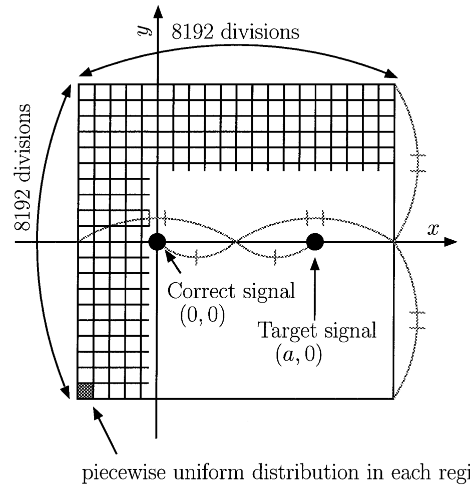

Fig. 4 Approximation of simulation probability density qq function.

the original p.d.f. Conversely, the simulation time to achieve a certain variance increases when $\nu^* T \to \nu T$ and $\nu^* T \to 1$. Therefore, an optimal $\nu^* T$ seems to exist.

7. Numerical Example

Since it is very difficult to generate a random number with arbitrary p.d.f., the simulation p.d.f. is approximated by only the rectangular region so that it includes the correct signal and the target error signal (Fig. 4). Furthermore, it is divided into 8192×8192 small rectangular regions. In each small region, the simulation p.d.f. is approximated by the uniform distribution.

As the encoder, the $(11,2,4)_8$ Ungerboeck code [7] with a $2/3$ rate was used. The natural mapping of 8-PSK was used for signal mapping. The squared Euclidean distance was used as the branch metric [2]. Fifty representative error vector sequences were selected from those with the smallest average squared Euclidean distance. The number of simulation runs for each representative error vector sequence was 5000. As the parameters of the lognormal noise, $\nu T = 0.01$ and $B = 2$ were used. The parameter $\nu^* T$ in the simulation p.d.f. was 0.01, 0.2, 0.4, and 0.6. The rectangular region in the approximated simulation p.d.f. was selected so that the probability of the region of the simulation p.d.f. is more than 0.99. The ordinary MC method was used for comparison with the IS simulation. The simulation continued until 200 error bits were observed.

Figure 5 shows the BER performance. The estimators obtained by the IS method are almost the same as those obtained by the MC method. The proposed method can be used to estimate the BER, and the number of representative error vector sequences is sufficient to evaluate the BER in this example.

The simulation time required to attain the same estimator accuracy should be considered to clarify the

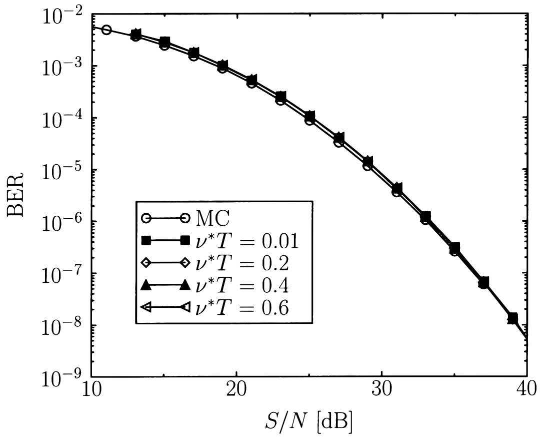

Fig. 5 BER performance.

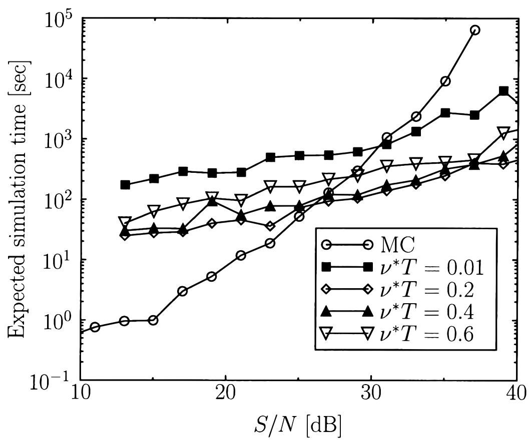

Fig. 6 Expected simulation time required to attain coefficient of variation $C_V = 10^{-1}$.

effectiveness of the proposed method. As the measure of the accuracy, the coefficient of variation C_V is used. It is defined as

$$C_V = \frac{\sqrt{\frac{1}{L}\sum_{i=1}^{L}\left\{\widehat{P}^{(i)} - \left(\frac{1}{L}\sum_{j=1}^{L}\widehat{P}^{(j)}\right)\right\}^2}}{\frac{1}{L}\sum_{i=1}^{L}\widehat{P}^{(i)}}, \quad (16)$$

where $\widehat{P}^{(i)}$ is the ith sample. From the above definition, the coefficient of variation C_V is the standard deviation to mean ratio. The expected simulation time required to attain $C_V = 10^{-1}$ is shown in Fig. 6. The number of samples in the numerical example was one hundred. From Fig. 6, the expected simulation time at $\nu^*T = 0.2$ and 0.4 is the quickest to estimate the BER. To evaluate the estimator with the same coefficient of variation, the simulation time using IS is about $1/10$ that using the MC method at BER $= 10^{-6}$ ($S/N \approx 33$ [dB]). The optimal value of ν^*T in this example is around $\nu^*T = 0.2$ to 0.4. The higher the S/N, the larger the difference of the simulation time between the MC and IS methods. The expected simulation time with the same coefficient of variation is not sensitive to ν^*T since the simulation time required to attain the same coefficient of variation is almost constant at $\nu^*T = 0.2$ and 0.4. At $\nu^*T = 0.01$ (i.e., the speed up parameter is equal to the original probability of the impulsive noise), the expected simulation time is almost the same as that for the MC method.

8. Conclusion

We have proposed a design method of the simulation probability density function for the impulsive noise in Refs. [1], [2] for the trellis-coded modulation scheme. The simulation time was reduced to about $1/10$ in the estimation of the bit error rate of 10^{-6} with the same coefficient of variation when the parameter of impulsive noise was $\nu^*T = 0.2$ and 0.4.

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