Chapter 2

Minimum Energy Coding

2.1 Introduction

Nodes in wireless networks are usually deployed forming a very dense network in which few meters is the typical distance between them. The communication of one with each other can cause serious problems since nearby nodes can overwhelm (MAI) the received signal of the desired user. CDMA is a promising multiple access scheme for sensor and ad hoc networks due to its interference averaging properties [10]. However, the performance of CDMA systems is limited by MAI.

In the past decade numerous methods have been developed to reduce MAI, most of which focus on the design of effective correlation receivers. However, they also introduce an increase in complexity, and often, also in the demand of computational power, something which is an undesired effect. In this section a different approach focusing on source coding is studied. Instead of merely designing receivers to suppress interferences the output of the source is represented with a special codebook so that MAI is greatly reduced.

In the remaining sections we start by simply introducing the On-Off Keying modulation scheme and the ME coding to finalize with a further analysis on the system performance. This is done by first introducing the signal model of the system to subsequently analyze different relevant performance parameters.

2.2 On-Off Keying modulation scheme (OOK)

On-off keying is a basic type of modulation that represents digital data as the presence or absence of a carrier wave. In its most basic form, the presence of a carrier during the bit duration represents a binary one (i.e., 1), while a binary zero (i.e., 0) results in no signal being transmitted. This modulation technique yields not a very efficient use of the spectrum due to the abrupt changes in amplitude of the carrier wave. Having a look at its power consumption properties, however, it can be appreciated that its performance is better than that of BPSK, for instance, due to the fact that energy consumption is larger when high bits are transmitted than when low ones are. In Figure 2.1 the basic working of BPSK and OOK is depicted.

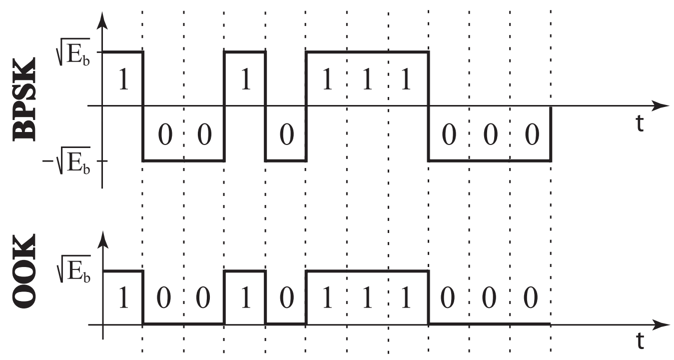


Figure 2.1: *BPSK and OOK modulation schemes.*

2.3 Energy consumption in OOK modulation

During the operation of a sensor node, power is consumed in sensing, data processing, data storing and communicating [11][12]. Among the four domains we focus in the communications one, since it is the most power consuming. The average energy consumption of a pair of nodes, one transmitting and one receiving, in a OOK modulation can be generally modelled as

[12]:

$$\begin{aligned} E_{radio} &= \widetilde{E}_{tx} + \widetilde{E}_{rx} \\ &= P_{tx,ckt}\left(T_{on,tx} + T_{startup}\right) + \alpha P_t T_{on,tx} + E_{dsp}^{(e)} \\ &\quad + P_{rx,ckt}\left(T_{on,rx} + T_{startup}\right) + E_{dsp}^{(d)} \end{aligned}$$

where $\widetilde{E}_{tx/rx}$ is the average energy consumption of a sensor node while transmitting/receiving; $P_{tx/rx,ckt}$ is the power consumption of the electronic circuits while transmitting/receiving; P_t is the output transmission power; $T_{on,tx/rx}$ is the transmitter/receiver ontime, and $T_{startup}$ is the start up time of the transceiver; $E_{dsp}^{(e/d)}$ is the energy consumed by the circuitry in encoding/decoding the data. In general, the energy spent in encoding/decoding is negligible compared to that needed to transmit and receive. Since $P_{tx/rx,ckt}$ and $T_{startup}$ are hardware dependent, these parameters cannot be used for the purpose of reducing power consumption by means of coding the source outputs. In the above expression we have also modelled the major characteristic brought about by the OOK modulation: the effective transmitting time is only a fraction of the transmitter ontime (where α is precisely that fraction of time).

2.4 The ME coding

Minimum Energy (ME) coding [13] attempts to optimize the power consumption in digital RF transmitters, which constitutes one of the most power consuming sources in portable communication devices. The function of a digital RF transmitter is to convert the modulated binary codewords into radio frequency waves able to travel through the air to reach other communication devices. The power needed to generate these signals is one of the major sources of power consumption in sensor nodes.

Any attempt to formulate the power optimization problem must be based on a deep understanding on how these waves are generated, therefore the type of modulation used is of extreme importance. For the application of ME coding, On-Off Keying (OOK) modulation is considered. Despite his limited performance when compared to other modulation schemes like

BPSK (it performs nominally 3dB worse due to reduced minimum distance in the signal constellation), the gain obtained when used along with ME is more than sufficient to justify its presence.

In Eq. (2.1) the power consumed in a WSN that uses ME coding is shown. The main characteristic of ME coding is that it reduces the value of the α coefficient (now termed α_{ME}).

$$\begin{aligned} E_{radio} &= \widetilde{E}_{tx} + \widetilde{E}_{rx} \\ &= P_{tx,ckt}\left(T^{ME}_{on,tx} + T_{startup}\right) + \alpha_{ME} P_t T^{ME}_{on,tx} + E^{(e)}_{dsp} \\ &\quad + P_{rx,ckt}\left(T^{ME}_{on,rx} + T_{startup}\right) + E^{(d)}_{dsp} \end{aligned} \tag{2.1}$$

Two key aspects should be considered now: with ME coding we are increasing the value of two system parameters, $T^{ME}_{on,tx/rx}$ and the length of the codeword L_{ME}.

- Increasing the transmitter ontime is not a disadvantage, since the major power consumption in modern low-power chips is given by P_t ($P_t >> P_{tx,ckt}$) and this term is affected by the design parameter $\alpha_{ME} << 1$. However, increasing the receiver ontime is extremely harmful because in the typical distances characteristic of a WSN, the power spent in receiving is approximately the same than that used in transmitting, so it could happen that no power savings at all are achieved. Subsection 2.8.2.2 deals with the evaluation of the ME coding power consumption properties. In order to reduce the $T_{on,rx}$ we will investigate a new coding scheme in Chapter 3.
- On the other hand, an increased codeword length could also be fatal, since we would increase the codeword error probability. However, the reduction in the multiaccess interference (provoked by the decreased number of high bits achieved by the ME coding) is more than enough to compensate this drawback.

Equation (2.1) suggests several ways of reducing the power consumption. Power consumption can be optimized by (i) minimizing $P_{tx,ckt}$ and P_t, which is done by improving the transmitter circuitry, (ii) reducing the

$T_{on,tx/rx}$, that can be done modifying the bit period, T_b (this results in an enlarged frequency spectrum) and, (iii) minimizing the presence of high bits in the codebook.

The objective of ME coding is to reduce the proportion of high bits in the codebook, α_{ME}. There are two ways of doing this:

- Use a set of codewords with a lesser number of high bits than the one used previously.
- Exploit the statistics of the source to assign codewords with a smaller number of high bits to the most probable symbols.

ME coding combines this two methods to provide the energy-optimal coding algorithm, which, based on the two previous bullets is constructed in two steps: Codebook Optimality and Coding Optimality. The former is to determine a set of codewords, termed a codebook, that has the fewest high bits, and the latter is to assign codewords having less high bits to messages with higher probability.

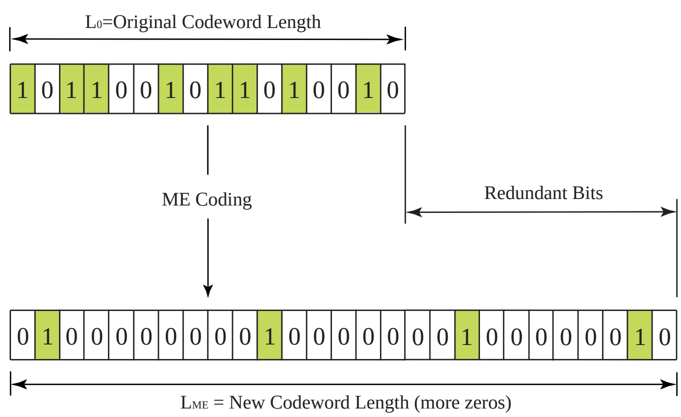


Figure 2.2: *Principle of Minimum Energy Coding.*

Basically, with ME coding we perform a mapping from an original set of codewords to another one more suitable to the aim of power savings. Each codeword in the original set has a unique image codeword in the new codebook. Specifically, the source codewords have length of L_0 bits and the ME codewords have length L_{ME} bits, with $L_{ME} > L_0$. Thus, what ME coding does is nothing different from endowing the source codebook with new codewords having less high bits. As a result, the new codebook has larger codewords, but we do not need to use all the possible new codewords, in fact, we will just make use of those more suitable for our design goal. Figure 2.3 illustrates this concept.

Thanks to the increase of the codeword length, we will be able to allocate new codewords with a smaller number of high bits to the original codewords, reducing in this way the probability of high bit, α_{ME}, and achieving power savings by means of OOK as previously explained.

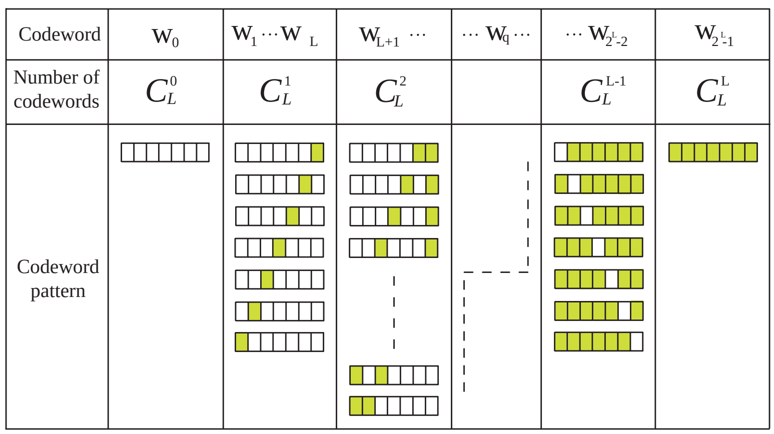


Figure 2.3: *Fixed-Length ME Codewords.*

In one sentence, ME Coding consist of: *assigning q codewords of the minimum codebook in the ascending order of number of high bits to the q-messages in the descending order of message probabilities*.

Of practical importance is ME Coding with fixed length codewords. It is clear that, for a q-codeword codebook, as the codeword length L_{ME} becomes longer, the number of high bits decreases. An extreme case is the *unary coding*, extensively studied in [14], which presents an interesting behavior. Unary coding uses a codeword length, L_{ME}, long enough to express all q messages with at most one high bit per codeword.

2.5 MAI reduction by ME source coding

The ME coding described above is useful for the reduction of MAI when using DS-CDMA. This is attained thanks to the low probability of overlap between the signals belonging to different users when we have a scenario consisting of multiple transmitters and receivers (see Figure 2.4). Since ME helps to decrease the number of high bits in the codewords it decreases the probability that two or more users transmit a high bit in the same time, achieving a reduction in the MAI.

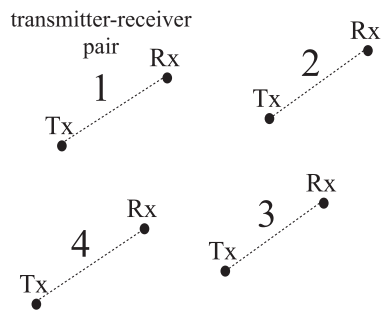
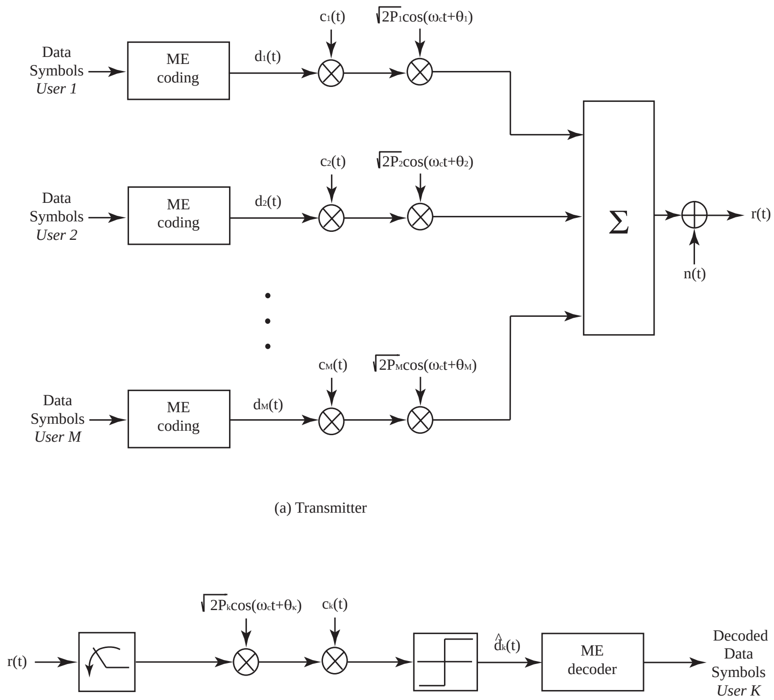


Figure 2.4: *System scenario.*

In the literature several methods for reducing MAI have been proposed. When dealing with MAI in the transmitter side, increasing the processing gain and boosting the signal are the two most common choices, but none of them are suitable for our purpose. The first one brings an undesired increase of the complexity of the receiver, while the second one moves in the opposite direction of the policy of power saving we are adopting.

Trying to reduce MAI in the receiver leads to sophisticated correlation filters that increase the complexity and cost of the design. The salient characteristic of ME coding is that the more power it saves, the smaller MAI it reaches, however, at the expense of sacrificing either transmission rate (this is not a burden in WSN where we are supposed to transmit at a low rate) or transmission time.

Figure 2.5 shows the configuration of the transmitter in our system when we use the source coding technique proposed. It is a quasi-typical DS-CDMA system in which, prior to multiplication by the spreading sequence, we perform a mapping from the source symbols to ME codewords. At the receiver the structure is similar but inverse. We say it is a *quasi*-typical DS-CDMA system because the BPSK modulator has been substituted by an On-Off Keying.



(a) Transmitter

(b) Receiver

Figure 2.5: *DS-CDMA combined with ME source coding.*

The proposed scheme causes a superimposition of the signals originated in the RF transmitters and, although the DS-CDMA system could send multiple non-zero signals at the same time (unique PN sequences are assigned to different users), with the new source coding technique this chance is reduced, and hence, the performance of the system increased (due to a lowered MAI). It should be noted that as the number of wireless sensors increases, so do the interferences. This fact can be overcome by simply enlarging the codewords (individual users will have sparser non-zero signals).

2.6 Signal model

In this section we introduce the signal model of the system with which we are working. We are dealing with an asynchronous DS-CDMA system and a wireless channel where we assume we have K active transmitter-receiver pairs of nodes carrying a bit rate of $R_b = \frac{1}{T_b}$. The same fixed bandwidth W, and hence the same chip interval T_c, is allocated to each channel. Let $a_k(t)$ and $b_k(t)$ denote the spreading signal and the baseband data signal, respectively, for pair k. Each one of these signals can be expressed as:

$$a_k(t) = \sum_{l=-\infty}^{\infty} a_l^{(k)} p_{T_c}(t - lT_c) \tag{2.2}$$

$$b_k(t) = \sum_{l=-\infty}^{\infty} b_{k,l} p_{T_b}(t - lT_b) \tag{2.3}$$

Where $p_{T_c}(t)$ is a rectangular pulse which takes a constant amplitude of value one from $t = 0$ to $t = T_c$ and zero outside this interval. $p_{T_b}(t)$ is similar to $p_{T_c}(t)$ but with a time duration of T_b.

$\{a_l^{(k)}\}_{l=0}^{\infty}$ and $\{b_{k,l}\}_{l=0}^{\infty}$ are the binary sequences corresponding to the spreading sequence and baseband data signal, respectively, for user k.

In this situation, the signal at the transmitter side of the pair k can be formulated as:

$$s_k(t) = \sqrt{2P_k} b_k(t) a_k(t) \cos(2\pi f_c t + \theta_k) \tag{2.4}$$

Where P_k denotes the signal power of the transmitter node in link k, f_c is the carrier frequency and θ_k is the carrier phase.

If we now want to represent the signal at the input of the receiver in pair i, $r_i(t)$, we have to realize that it is composed by several different terms: the desired signal, the signals transmitted by other users and a noise component introduced by the AWGN channel. From now on we will consider user i, denoting it with the correspondent subscript:

$$r_i(t) = \sum_{k=1}^{K} \sqrt{2P_k \Omega_{ki}} b_k(t - \tau_k) a_k(t - \tau_k) \cos(2\pi f_c t + \psi_k) + n(t). \tag{2.5}$$

Where τ_k stands for the signal delay for the k^{th} user, $\psi_k = \theta_k - 2\pi f_c \tau_k$ and $n(t)$ is the gaussian noise with two sided spectral density $\frac{N_0}{2}$. We have also introduced the wireless channel influence by means of the channel coefficients, Ω_{ki}, which are defined in the following way

$$\Omega_{ki} = \mathrm{PL}_{ki} e^{\xi_{ki}} \tag{2.6}$$

where PL_{ki} is the loss present in the path from the transmitter in pair k to the receiver in pair i, and ξ_{ki} is the shadow fading parameter. The path loss can be further written (in dBs) as

$$\mathrm{PL}_{ki}|_{dB} = -P_l\left(d_r\right)|_{dB} - 10n\log_{10}\left(\frac{d_{ki}}{d_r}\right)$$

with the reference distance $d_r = \text{constant} = 1$ meter. d_{ki} is the distance between the transmitter in channel k and the receiver in channel i, $P_l\left(d_r\right)|_{dB}$ is a known constant (55 dB for the Telos motes), and n is the path-loss decay constant, which takes the value 2 for the free space and $[3.5 - 4]$ for urban environments. The shadowing component of Ω_{ki} is described by a log-normal distribution $e^{\xi_{ki}}$, where ξ_{ki} is a zero-mean Gaussian distributed r.v. with variance $\sigma^2_{\xi_{ki}}$.

If the DS-CDMA system were completely synchronized, then one could ignore the time delays τ_k $(k = 1, 2, \cdots, K)$. Of course, this would require a perfect common timing reference for the K transmitters being necessary to introduce mechanisms for compensating the delays in the various transmission paths. However, this is not easy to implement, so that the asynchronous system is the one usually implemented.

Let us consider channel i. Since we are interested in relative phase shifts modulo 2π, there is no loss of generality in assuming $\theta_i = 0$ and $\tau_i = 0$. Following an approach similar to that found in [15] we can express the output of the correlation receiver matched to $s_i(t)$ as:

$$Z_i = \int_0^{T_b} r(t) a_i(t) \cos\left(2\pi f_c t\right) dt \tag{2.7}$$

From now on, we assume that $f_c >> T_b^{-1}$. This is a realistic assumption that helps us to ignore the double frequency component of $r(t)\cos\left(2\pi f_c t\right)$ that appears when calculating the output of the correlation receiver. Thus, it can be shown that Eq. (2.7) can be expressed as:

$$Z_i = D_i + I_i + N_g \tag{2.8}$$

where D_i is the desired signal in the channel i, I_i is the interference term due to the presence of multiple users (MAI) and N_g is a Gaussian random variable with zero mean and variance $\frac{N_0 T_b}{4}$.

$$\begin{aligned} D_i &= \sqrt{\frac{P_i \Omega_{ii}}{2}} T_b b_{i,0} && (2.9) \\ I_i &= \sum_{\substack{k=1 \\ k \neq i}}^{K} \sqrt{\frac{P_k \Omega_{ki}}{2}} \cos(\psi_k) \, B(i, k, \tau_k) && (2.10) \\ N_g &= \int_0^{T_b} n(t) a_i(t) \cos(2\pi f_c t) \, dt && (2.11) \end{aligned}$$

Without loss of generality we observe the output of the correlation receiver at the first time instant ($l = 0$). Also, for convenience, we have introduced the term $B(i, k, \tau)$ adopting a simplified version of that presented in [16] and used in [17].

$$B(i, k, \tau) = \int_0^{T_b} b_k (t - \tau) \, a_k (t - \tau) \, a_i (t) \, dt \qquad (2.12)$$

2.7 Signal to Interference and Noise Ratio (SINR)

Once we have the signal model perfectly determined, we can undertake the task of calculating the Signal to Interference and Noise Ratio ($SINR$) parameter, which is one of the most important performance measures and can be obtained within a reasonable computational complexity. The $SINR$ for transmitter-receiver pair n (denoted as $SINR_n$) is then defined as the ratio between the average and the standard deviation of Z_i (2.13), the expectation being taken with respect to carrier phases, time delays and data symbols, but not with respect to the channel coefficients (which are supposed to be slow variant and constant for a period around T_b). However, we have to make a distinction in the computation of the $SINR$ depending on what bit is transmitted:

1. If $\mathbf{b_{i,0} = 0}$, since we do not transmit anything (recall we are using OOK), the power of the desired signal is zero, and hence, the $SINR|_{dB} = -\infty$.

2. If $\mathbf{b_{i,0} = 1}$, the calculation of the $SINR$ can be made via Eq. (2.13). From now on, let us consider the pair $n = i$:

$$SINR_i = \frac{E\left[Z_i\right]}{\sqrt{Var[N_g + I_i]}} \tag{2.13}$$

Since the gaussian noise and the MAI are independent, we can write:

$$Var[N_g + I_i] = Var[N_g] + Var[I_i]$$

Thus, the power of the gaussian component of the noise can be calculated as:

$$\begin{aligned} Var[N_g] &= Var\left[\int_0^{T_b} n(t)a_i(t)\cos\left(2\pi f_c t\right) dt\right] \\ &= \int_0^{T_b} Var\left[n(t)\right] a_i^2(t)\cos^2\left(2\pi f_c t\right) dt \\ &= \frac{N_0 T_b}{4} \end{aligned} \tag{2.14}$$

In all that follows we consider the time delays, phase shifts and data symbols $(\psi_k, \tau_k, b_{k,l})$ as mutually independent random variables. Namely, we treat ψ_k and τ_k as two r.v. uniformly distributed in their respective range of values, $[0, 2\pi]$ and $[0, T_b]$. Also, we assume that the data symbols $b_{k,l}$ take values $\{0, 1\}$ with a determinate probability $\{1 - \alpha_{ME,k}, \alpha_{ME,k}\}$. Independence among the different transmitting nodes is a realistic assumption too. All these assumptions are representative of real systems [17].

With all these hypotheses in mind, it can be proved that $E\left[B\left(i,k,\tau\right)\right] = 0$, $Var\left[B\left(i,k,\tau\right)\right] \cong \frac{2T_b^2}{3G}$ and $E\left[\cos^2\left(\psi_k\right)\right] = \frac{1}{2}$ where $G = \frac{W}{R_b} = \frac{T_b}{T_c}$ denotes the processing gain.

Therefore, assuming $\alpha_{ME,1} = \alpha_{ME,2} = \cdots = \alpha_{ME,K} = \alpha_{ME}$ and taking into account all previous considerations and Eq. (2.13), we can rewrite the $SINR_i$ as:

$$SINR_i = \frac{\sqrt{\frac{P_i \Omega_{ii}}{2}} T_b}{\left(\frac{N_0 T_b}{4} + Var\left[I_i\right]\right)^{\frac{1}{2}}} \tag{2.15}$$

The expression of the variance $Var\left[I_k\right]$ turns out to be:

$$Var\left[I_k\right] \cong \sum_{\substack{j=1 \\ j\neq k}}^{K} \alpha_{ME} P_j \Omega_{jk} \frac{T_b^2}{6G} \tag{2.16}$$

Please note the inclusion of the α_{ME} coefficient in the formula above (2.16). This is done because not all the remaining nodes may be transmitting a high bit when the user in channel i does. Taking into account that α_{ME} is the probability of sending a high bit, on average, only a α_{ME} fraction of the codeword time will be carrying ones.

If we introduce Eq. (2.16) into the $SINR_i$ we arrive at:

$$SINR_i \cong \sqrt{\frac{\frac{P_i \Omega_{ii}}{2} T_b^2}{\frac{N_0 T_b}{4} + \sum\limits_{\substack{j=1 \\ j\neq i}}^{K} \alpha_{ME} P_j \Omega_{ji} \frac{T_b^2}{6G}}} \tag{2.17}$$

2.8 Power consumption

In this section we investigate the power consumption which takes place in the network when the ME coding is used. To start with, we solve a constrained optimization problem to subsequently evaluate the performance of the ME coding.

2.8.1 Optimal transmission power

Let us imagine a system in which we have K transmitter-receiver pairs. In this scenario (see figure 2.4), if we want to optimize the overall network power consumption, we have to consider the transmission power in each link. However, minimizing the transmission power requires to take into account the resulting effect on other system parameters as the probability of error, $SINR$...because they are all interrelated. Thus, we investigate the optimization problem:

$$\min_{\mathbf{P}} \sum_{i=1}^{K} P_i \tag{2.18}$$

$$s.t. \quad P\left[SINR_i \leq \gamma\right] \leq \bar{P}_{out} \;, \forall\, i = 1...K \tag{2.19}$$

$$P_i > 0 \quad \forall\, i = 1, \ldots, K$$

where $\mathbf{P}$ denotes the vector composed by the transmission powers of the different nodes:

$$\mathbf{P} = \left[P_1, \ldots, P_i, \ldots, P_K\right]^T \tag{2.20}$$

In order to minimize the transmission power of the overall system, we propose an optimization problem whose objective function is the sum of the powers of all the transmit nodes, while the constraints are expressed in terms of link outage probability [18].

In the optimization problem, note that γ is defined as the SINR threshold for the computation of the outage probability. In particular, the solution of the optimization problem ensures that the outage probability remains below the maximum value $\bar{P}_{out}$. Furthermore, note that the computation of the outage probability is performed with respect to the statistics of the wireless channel, and the distribution of high bits.

To solve the optimization problem (2.18), we have to model the constraints related to the outage probabilities of the links. Since the statistics of the SINR are in general unknown, we resort to the well know extended Wilkinson moment-matching method [17]. Specifically, we approximate the SINR with an overall Log-normal distribution, thus obtaining that

$$SINR_i = A_i \triangleq L_i^{-\frac{1}{2}} \approx e^{-\frac{1}{2}x_i} \tag{2.21}$$

and

$$L_i^{-1} = \frac{\frac{P_i \Omega_{ii} T_b^2}{2}}{\frac{N_0 T_b}{4} + Var[I_i]} \tag{2.22}$$

where it can be proved that $x_i \sim \mathcal{N}\left(\mu_{x_i}, \sigma_{x_i}\right)$ [17]. The approximation is useful because allows for computing the expression of the outage probability while taking into account all the relevant aspects of the wireless propagation, the transmission power, and the coding statistics. It trivially results that

$$P\left[SINR_i \leq \gamma\right] \approx P\left[e^{-\frac{x_i}{2}} \leq \gamma\right] = Q\left(\frac{-2\ln\gamma - \mu_{x_i}}{\sigma_{x_i}}\right) \tag{2.23}$$

Let us calculate the mean and variance of A_i [19],

$$\mu_{A_i} = E_{\mathbf{\Omega}_i}\left[A_i\right] = e^{-\frac{\mu_{x_i}}{2}+\frac{\sigma^2_{x_i}}{8}} \tag{2.24}$$

$$r_{A_i} = E_{\mathbf{\Omega}_i}\left[A_i^2\right] = e^{-\mu_{x_i}+\frac{\sigma^2_{x_i}}{2}} \tag{2.25}$$

$$\sigma^2_{A_i} = r_{A_i} - \mu^2_{A_i} \tag{2.26}$$

It can be shown that

$$\mu_{x_i} = 2\ln M_1^i - \frac{1}{2}\ln M_2^i \tag{2.27}$$

$$\sigma^2_{x_i} = \ln M_2^i - 2\ln M_1^i \tag{2.28}$$

where

$$M_1^i \triangleq E_{\mathbf{\Omega}_i}\left[L_i\right] \tag{2.29}$$

$$M_2^i \triangleq E_{\mathbf{\Omega}_i}\left[L_i^2\right] \tag{2.30}$$

If we recall the definition of L_i,

$$L_i = \frac{2}{P_i T_b^2 \mathrm{PL}_{ii}}\left(\sum_{\substack{j=1\\ j\neq i}}^{K} \alpha_{ME} P_j \mathrm{PL}_{ji} e^{\xi_{ji}-\xi_{ii}} \frac{T_b^2}{6G} + \frac{N_0 T_b}{4} e^{-\xi_{ii}}\right) \tag{2.31}$$

We can easily calculate the value of M_1^i and M_2^i applying the statistical expectation operator to L_i and L_i^2.

$$M_1^i = \frac{2}{P_i T_b^2 \mathrm{PL}_{ii}}\beta_1^i \tag{2.32}$$

$$M_2^i = \frac{4}{P_i^2 T_b^4 \mathrm{PL}_{ii}^2}\beta_2^i \tag{2.33}$$

where the β coefficients are given by

$$\beta_1^i = \sum_{\substack{j=1 \\ j\neq i}}^{K} \alpha_{ME} P_j \mathrm{PL}_{ji} e^{\mu_{\xi_{ji}}-\mu_{\xi_{ii}}+\frac{1}{2}\left(\sigma^2_{\xi_{ji}}+\sigma^2_{\xi_{ii}}\right)} \frac{T_b^2}{6G} + \frac{N_0 T_b}{4} e^{-\mu_{\xi_{ii}}+\frac{\sigma^2_{\xi_{ii}}}{2}} \tag{2.34}$$

$$\begin{aligned}\beta_2^i = & \sum_{\substack{j=1 \\ j\neq i}}^{K} \alpha_{ME}^2 P_j^2 \mathrm{PL}_{ji}^2 e^{2\left(\mu_{\xi_{ji}}-\mu_{\xi_{ii}}+\sigma^2_{\xi_{ji}}+\sigma^2_{\xi_{ii}}\right)} \frac{T_b^4}{36G^2} + \frac{N_0^2 T_b^2}{16} e^{2\left(-\mu_{\xi_{ii}}+\sigma^2_{\xi_{ii}}\right)} \\ & + \sum_{\substack{j=1 \\ j\neq i}}^{K} \sum_{\substack{k=1 \\ k\neq i \\ k\neq j}}^{K} \alpha_{ME}^2 P_j P_k \mathrm{PL}_{ji} \mathrm{PL}_{ki} e^{\left(\mu_{\xi_{ji}}+\mu_{\xi_{ki}}-2\mu_{\xi_{ii}}\right)+\frac{1}{2}\left(\sigma^2_{\xi_{ji}}+\sigma^2_{\xi_{ki}}+4\sigma^2_{\xi_{ii}}\right)} \frac{T_b^4}{36G^2} \\ & + \frac{N_0 T_b}{2} \sum_{\substack{j=1 \\ j\neq i}}^{K} \alpha_{ME} P_j \mathrm{PL}_{ji} e^{\mu_{\xi_{ji}}-2\mu_{\xi_{ii}}+\frac{1}{2}\left(\sigma^2_{\xi_{ji}}+4\sigma^2_{\xi_{ii}}\right)} \frac{T_b^2}{6G}\end{aligned} \tag{2.35}$$

It should be noticed that neither in β_1^i nor in β_2^i there is dependence with the transmission power in channel i (P_i).

The constraint on the outage probability can be rewritten in order to evidence the dependence on the transmission power coefficients. After some algebra, a relaxation of the minimization program can be expressed as:

$$\min_{\mathbf{P}} \sum_{i=1}^{K} P_i \tag{2.36}$$

$$s.t. \quad \frac{P_i}{2T_b^{-2}\mathrm{PL}_{ii}^{-1}\left(\beta_1^i\right)^{2\left(1-Q^{-1}\left(\bar{P}_{out}\right)\right)}\left(\beta_2^i\right)^{-\frac{1}{2}+Q^{-1}\left(\bar{P}_{out}\right)}} \geq \gamma^2, \quad i = 1 \ldots K$$

$$P_i > 0 \quad \forall\, i = 1, \ldots, K \tag{2.37}$$

The problem (2.36) is a relaxation since σ_{x_i} has been replaced with its square. This is equivalent to say that the expectations are tighter. It is possible to see that the relaxation reduces the computational burden, and that the solution is an upper bound of the solution of the original problem. The program (2.36), is a centralized problem, in the sense that to compute the solution, a central node should be able to collect all the information related to radio link coefficients, it should be able to solve the program, and finally it should broadcast the optimized powers to all other nodes. A centralize implementation exhibits clear disadvantages in terms of communication resources.

Nevertheless, it can be proved that (2.36) can be solved with a fully distributed strategy. Specifically, by following the same approach proposed in [20], each receiver node can find iteratively the optimal power as follows:

$$P_i(n) = \gamma^2 \left(2T_b^{-2} \mathrm{PL}_{ii}^{-1} \left(\beta_1^i\right)^{2\left(1-Q^{-1}\left(\bar{P}_{out}\right)\right)} \left(\beta_2^i\right)^{-\frac{1}{2}+Q^{-1}\left(\bar{P}_{out}\right)} \right)^{\{n-1\}} \tag{2.38}$$

The power updating can be done asynchronously by each node, and it can be proved that for $n \to \infty$ (n denotes time) the power converges to the optimal value [21]. The algorithm (2.38) is fully distributed, since the computation of the path loss parameter, as well as the beta coefficients, is done locally by the nodes. In particular, note hat the node i does not need to know the powers of the other nodes, but it has just to compute the expectations β_i^1 and β_i^2, through (2.32) and (2.33).

2.8.2 Numerical results

2.8.2.1 Power minimization algorithm

In this section, a numerical implementation is derived and discussed. We consider the same scenario addressed in [22] and in [18]. We consider the existence of $K = 10$ different pairs, each one with their respective transmitter and receiver. All nodes have been placed randomly within a distance of 3 to 15 meters between each other. Furthermore, we assume that $R_b = 250$ Kbps, the processing gain $G = 64$ and the path-loss decay constant $n = 4$; the power spectral density of the gaussian noise is -174 dBm, $\alpha_{ME} = 0.1$ and the expected value of the signal to interference + noise ratio threshold is set to be $E_{\mathbf{\Omega_i}}\left[SINR_i\right]_{dB} \geq 3.1$ dB ($i = 1..K$). The chosen probability of outage is the 1%.

To find suitable values for $\mu_{\xi_{ji}}$ and $\sigma^2_{\xi_{ji}}$, $i, j = 1..K$, we established a comparison between our model for the path-losses (2.6) and that found in [23][22]:

$$\Omega_{ki}|_{dB} = -P_l\left(d_r\right)|_{dB} - 10n\log_{10}\left(\frac{d_{ki}}{d_r}\right) - X_\sigma|_{dB} \tag{2.39}$$

where $n = 4$ and $X_\sigma|_{dB}$ has been shown to be a zero-mean Gaussian r.v. (in dB) with standard deviation $\sigma = 5$ representing the shadowing effects[1]. If

[1] n and σ were obtained through curve fitting of empirical data [22]

we rewrite our model (2.6) in decibels it appears to be:

$$\Omega_{ki}|_{dB} = -P_l\left(d_r\right)|_{dB} - 10n\log_{10}\left(\frac{d_{ki}}{d_r}\right) + 10\log_{10}\left(e\right)\xi_{ki} \tag{2.40}$$

Comparing both models we directly arrive at:

$$\xi_{ki} = \frac{-X_\sigma|_{dB}}{10\log_{10}\left(e\right)} \tag{2.41}$$

from where it is easy to derive

$$\mu_{\xi_{ki}} = 0 \tag{2.42}$$

$$\sigma^2_{\xi_{ki}} = \frac{\sigma^2}{\left(10\log_{10}\left(e\right)\right)^2} \tag{2.43}$$

For our simulation we assume that all links experience the same standard deviation of the slow fading (shadowing). Initially all nodes transmit at 0 dBm.

In Figure 2.6, the convergence of the limit (2.38) is shown. It can be appreciated how it barely takes 5 iterations to enter the stationary state. If we now

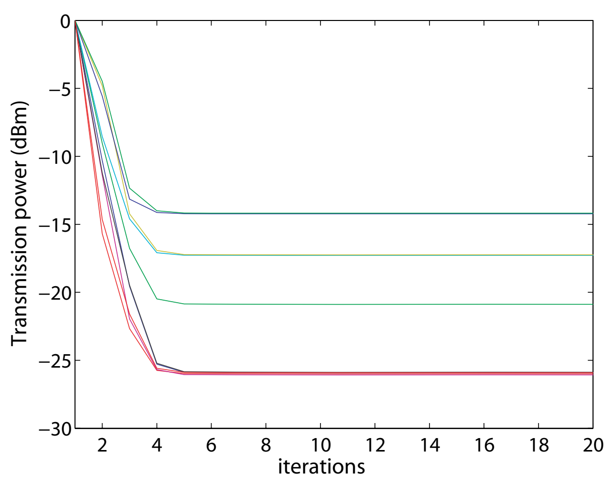


Figure 2.6: *Convergence of the power minimization algorithm.*

analyze the system we can observe that the mean value of the transmission power used in the network is -21.36 dBm and the achieved probability of outage is 0.0037, smaller than the 0.01 imposed.

2.8.2.2 Power consumption in ME coding

A comparison, in terms of total power consumption, between a typical BPSK system and a ME coding system is carried out in this subsection.

If we look thoroughly at the energy model for a system using ME coding (2.1) we can realize that, if we neglect the coding/decoding energies (as is usually done), is exactly the same than that of a BPSK system, except for the α_{ME} coefficient. We define the energy gain as the ratio of the energy used in a BPSK system and the energy used in a ME coding system.

$$\rho_{dB} = \left(\frac{E_{radio}^{BPSK}}{E_{radio}^{ME}} \right)_{dB} \tag{2.44}$$

To calculate the energy gain we have considered the CC2420 radio transceiver module by Chipcon, as is the one incorporated in the Telos motes. The values considered for the computation of the energy gain have been obtained from the CC2420 datasheet [24].

(a) $P_t = 0dBm$

α_{ME}	ρ_{dB}
0.1	2.46
0.2	2.11
0.3	1.78
0.4	1.48
0.5	1.19

(b) $P_t = -25dBm$

α_{ME}	ρ_{dB}
0.1	1.43
0.2	1.24
0.3	1.07
0.4	0.90
0.5	0.73

Table 2.1: *Energy gain of ME coding vs BPSK for two different transmission powers ($P_t = 0$ dBm and $P_t = -25$ dBm). The displayed gain corresponds to the converged value (the gain increases as the transmitting time does until it reaches a stable value).*

Two major conclusions can be drawn from Table 2.1. As we expected, the smaller the number of high bits in the ME codeword, the higher the gain. The second main result is that the higher the transmission power, the larger the gain, what is also logical, since this higher value of the transmission power allow us to further exploit the characteristics of the ME coding. This last statement is due to the value of $P_{tx/rx,ckt}$ compared to that of P_t. Thus, we need that P_t dominates over $P_{tx/rx,ckt}$ to take advantage of the use of ME coding. In this sense, ME coding will become more and more important in the future, as advances in electronics tend to reduce the power consumption of the circuitry.

2.9 Error probability

We can express the error probability for the pair i, P_e^i, as:

$$\begin{aligned} P_e^i &= \Pr(\text{Tx } 0)\Pr(Z_i > \delta_i | 0 \text{ Tx}) + \Pr(\text{Tx } 1)\Pr(Z_i < \delta_i | 1 \text{ Tx}) \\ &= (1-\alpha_{ME})\, p^i_{e|0} + \alpha_{ME} \cdot p^i_{e|1} \end{aligned} \quad (2.45)$$

where δ_i is known as the decision threshold for link i and we have defined, for convenience, the probabilities given by (2.46) and (2.47). From now on, we will indicate the dependence of the parameters with the considered transmitter-receiver pair, i, with the correspondent super/subscript.

$$p^i_{e|0} = \Pr(Z_i > \delta_i | 0 \text{ Tx}) \quad (2.46)$$

$$p^i_{e|1} = \Pr(Z_i < \delta_i | 1 \text{ Tx}) \quad (2.47)$$

For computing these probabilities let us distinguish, once more, between the two possible cases:

1. If $\mathbf{b_{i,0}} = 0$ the output of the matched filter at the receiver is formed exclusively by the noise component:

   $$Z_{i|0} = I_i + N_g$$

   The decision variable, $Z_{i|0}$, is given by a MAI term I_i and the thermal noise N_g.

   It can be assumed that I_i can be modelled as a Gaussian random variable with a distribution that is completely specified by its mean and variance, which is, in turn, a random variable [25] due to the wireless channel coefficients. Thus, $Z_{i|0}$, as the sum of two independent

Gaussian random variables, results in another Gaussian distributed random variable.

$$\begin{matrix} N_g \sim \mathcal{N}\left(0, \sqrt{\frac{N_0 T_b}{4}}\right) \\ I_i \sim \mathcal{N}\left(0, \sqrt{Var[I_i]}\right) \end{matrix} \rightarrow Z_{i|0} \sim \mathcal{N}\left(0, \sqrt{\frac{N_0 T_b}{4} + Var[I_i]}\right) \quad (2.48)$$

The previous considerations about the stochastic nature of $Z_{i|0}$, introduced by the channel coefficients, are of vital importance when calculating $p^i_{e|0}$. In fact, it is necessary to perform an average over the different realizations of the channel coefficients.

$$\begin{aligned} p^i_{e|0} &= \Pr\left(Z_{i|0} > \delta_i\right) \\ &= \Pr\left(I_i + N_g > \delta_i\right) \\ &= E_{\mathbf{\Omega}_i}\left[\Pr\left(I_i + N_g > \delta_i | \mathbf{\Omega}_i\right)\right] \end{aligned} \quad (2.49)$$

Let us calculate the probability of error for a single realization:

$$\begin{aligned} \Pr\left(I_i + N_g > \delta_i | \mathbf{\Omega}_i\right) &= \int_{\delta_i}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{Z_{i|0}}} e^{-\frac{t^2}{2\sigma^2_{Z_{i|0}}}} dt \\ &= \frac{1}{\sqrt{2\pi}} \int_{\frac{\delta_i}{\sigma_{Z_{i|0}}}}^{\infty} e^{-\frac{u^2}{2}} du \\ &= Q\left(\frac{\delta_i}{\sigma_{Z_{i|0}}}\right) \end{aligned}$$

where

$$Q\left(x\right) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{\frac{-u^2}{2}} du \quad \text{and} \quad \sigma_{Z_{i|0}} = \sqrt{\frac{N_0 T_b}{4} + Var[I_i]}$$

finally,

$$p^i_{e|0} = E_{\mathbf{\Omega}_i}\left[Q\left(\frac{\delta_i}{\sigma_{Z_{i|0}}}\right)\right] \quad (2.50)$$

2. If $\mathbf{b_{i,0}} = 1$, there is an additional term in the output of the correlation receiver correspondent to the power emitted when the high bit is transmitted:

$$Z_{i|1} = D_i + I_i + N_g$$

Again, $Z_{i|1}$ is a random variable due to both the MAI and the desired signal components of $Z_{i|1}$. As in the case of $b_{i,0} = 0$, $Z_{i|1}$ can be modelled as a Gaussian distributed random variable.

$$Z_{i|1} \sim \mathcal{N}\left(\mu_{Z_{i|1}}, \sigma_{Z_{i|1}}\right) = \mathcal{N}\left(\sqrt{\frac{P_i \Omega_{ii}}{2} T_b^2}, \sqrt{\frac{N_0 T_b}{4} + Var[I_i]}\right) \quad (2.51)$$

Once more we can write

$$\begin{aligned} p_{e|1}^i &= \Pr\left(Z_{i|1} < \delta_i\right) \\ &= \Pr\left(D_i + I_i + N_g < \delta_i\right) \\ &= E_{\mathbf{\Omega}_i}\left[\Pr\left(D_i + I_i + N_g < \delta_i | \mathbf{\Omega}_i\right)\right] \end{aligned} \quad (2.52)$$

Let us calculate the probability of error given the channel coefficients:

$$\begin{aligned} \Pr\left(D_i + I_i + N_g < \delta_i | \mathbf{\Omega}_i\right) &= 1 - \int_{\delta_i}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{Z_{i|1}}} e^{-\frac{\left(t-\mu_{Z_{i|1}}\right)^2}{2\sigma_{Z_{i|1}}^2}} dt \\ &= 1 - \frac{1}{\sqrt{2\pi}} \int_{\frac{\delta_i - \mu_{Z_{i|1}}}{\sigma_{Z_{i|1}}}}^{\infty} e^{-\frac{u^2}{2}} du \\ &= Q\left(\frac{\mu_{Z_{i|1}} - \delta_i}{\sigma_{Z_{i|1}}}\right) \end{aligned}$$

where the variance of $Z_{i|1}$ is mainly determined by the MAI term characterized by having fast fluctuations, in contrast with the desired signal which is slow changing (we can assume Ω_{ii} approximately constant during the bit duration). One can see that $\sigma_{Z_{i|1}} = \sigma_{Z_{i|0}} = \sigma_{Z_i}$, to finally write

$$p_{e|1}^i = E_{\mathbf{\Omega}_i}\left[Q\left(\frac{\mu_{Z_{i|1}} - \delta_i}{\sigma_{Z_i}}\right)\right] \quad (2.53)$$

If we recall the average bit error probability in the system (2.45), and make use of Eqs. (2.50) and (2.53), we can write that

$$\begin{aligned} P_e^i &= \left(1 - \alpha_{ME}\right) E_{\mathbf{\Omega}_i}\left[Q\left(\frac{\delta_i}{\sigma_{Z_i}}\right)\right] + \alpha_{ME} E_{\mathbf{\Omega}_i}\left[Q\left(\frac{\mu_{Z_{i|1}} - \delta_i}{\sigma_{Z_i}}\right)\right] \quad (2.54) \\ &= E_{\mathbf{\Omega}_i}\left[\left(1 - \alpha_{ME}\right) Q\left(\frac{\delta_i}{\sigma_{Z_i}}\right) + \alpha_{ME} Q\left(\frac{\mu_{Z_{i|1}} - \delta_i}{\sigma_{Z_i}}\right)\right] \quad (2.55) \end{aligned}$$

The probability (2.54) should be minimized with respect to the value of δ_i. Unfortunately, there is not a simple closed form solution for the value of δ_i, due to the non linear functions involved in the computation of the expectation in (2.54). Therefore, we resort to the heuristic

$$\delta_i = \frac{\mu_{Z_{i|1}}}{2} \tag{2.56}$$

Introducing this value into Eq. (2.55) and recalling the definition for the $SINR$ we obtain

$$P_e^i \ = \ E_{\mathbf{\Omega}_i}\left[Q\left(\frac{SINR_i}{2}\right)\right] \tag{2.57}$$

To evaluate the expression above (namely, the expectation of the Q function) we will make use of the Stirling Approximation, for what we need to calculate the mean and standard deviation of the argument of the Q function. Let us start writing the Stirling approximation for the expectation of the function Q with general argument ζ^i.

$$P_e^i \approx E_{\mathbf{\Omega}_i}\left[Q\left(\zeta^i\right)\right] \approx \frac{2}{3}Q\left(\mu_{\zeta^i}\right) + \frac{1}{6}Q\left(\mu_{\zeta^i} + \sqrt{3}\sigma_{\zeta^i}\right) + \frac{1}{6}Q\left(\mu_{\zeta^i} - \sqrt{3}\sigma_{\zeta^i}\right) \tag{2.58}$$

where μ_{ζ^i} and σ_{ζ^i} are the expectation and the standard deviation of ζ^i, respectively. We have defined

$$\begin{aligned}
\zeta_1^i &= \tfrac{SINR_i}{2} \\
\mu_{\zeta_1^i} &= \tfrac{1}{2}e^{-\frac{\mu_{x_i}}{2}+\frac{\sigma_{x_i}^2}{8}} \\
r_{\zeta_1^i} &= \tfrac{1}{4}e^{-\mu_{x_i}+\frac{\sigma_{x_i}^2}{2}} \\
\sigma_{\zeta_1^i}^2 &= r_{\zeta_1^i} - \mu_{\zeta_1^i}^2
\end{aligned}$$

Finally, it should be recalled that, in an interference limited system, the real bit error probability should be computed as

$$P_b^i = \bar{P}_{out}^i + \left(1 - \bar{P}_{out}^i\right) P_e^i \tag{2.59}$$

Figure 2.7 shows the bit error probability in the network ($K = 10$ pairs). For calculating this probability the wireless channel was taken into account. The power optimization algorithm was performed so we can positively state that the probability of error obtained is the minimum achievable for the given $SINR$. The parameters used for the wireless channel and the

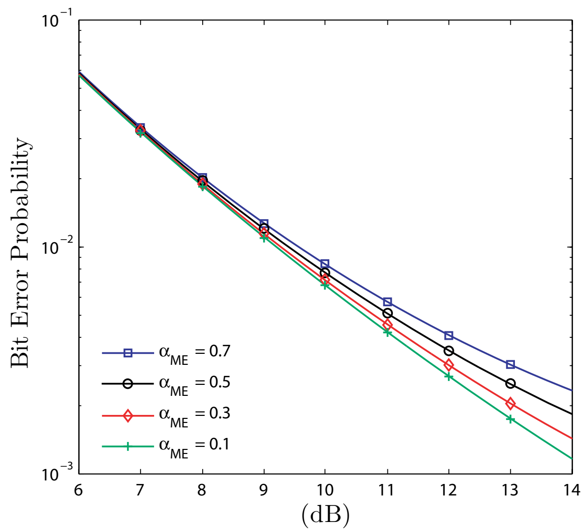


Figure 2.7: *Bit Error Probability with the variation of* α_{ME} *in a system using ME Coding when the wireless channel is considered and overall system power is minimized.* $R_b = 250$ *Kbps.*

power optimization algorithm were those presented in Subsection 2.8.2. In the x-axis we depict the average received $SINR$ (in decibels).

The main conclusion to be drawn from Figure 2.7 is that the larger the power savings (i.e, the smaller the α_{ME}), the lower the system bit error probability. This is obviously due to the fact that lower values of α decrease the multi access interference. However, ME coding is not a perfect system, and it also has undesirable effects, as are the increase in either the bandwidth requirements or in the transmission time. The former one is not a problem since bandwidth is not usually the major constraint in WSNs, but the latter could be a problem when running applications which involve the transmission of large amounts of data (which, fortunately, is not the usual case).