Chapter 4

Model of the Middle Ear

This chapter shows the model of the middle ear used in this job. As it was said before, Multibody Systems techniques were used. It is the objective of this chapter to make a description of the model.

4.1 Description of the System

According to the anatomy aspects showed in chapter 2, the ossicles are two bones: the stapes and the malleus-incus complex (MIC). The bodies were supposed as rigid bodies. This was a good approach since the bones were not going to suffer significant elastic deformations.

In anatomy, the connection between the ossicles are called joints (in the case of the GP the only joint is the incudo-stapedial joint) but mechanically they are coupling elements and were described in this model as visco-elastic coupling elements. Like the joints, the suspension of the ossicles in the tympanic membrane was considered as visco-elastic elements. At the malleus-incus complex were the: lig. mal. anterior, lig. mall. laterale, lig. inc. posterior and at the stapes the lig. annulare which sealed the inner ear. Two muscles were included in this model: the musculus tensor tympani and the musculus stapedius. Finally, the tympanic membrane was included in the model as a visco-elastic coupling element as well.

In this model they were defined 9 generalized coordinates. The orientation of the inertial frame was established in the usual way, taking as reference the geometry of the stapes: the y-axes in the piston-motion direction of the stapes, the x-axes in the direction of the long axe of the footplate and the z-axe perpendicular to both of them. The location and orientation of the inertial system is shown in the figure 4.1. All the local reference systems attached to the bones were defined in the same way.

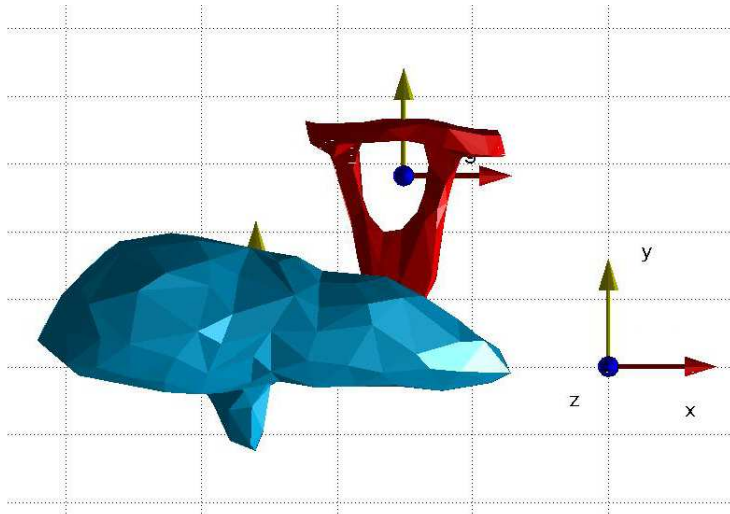


Figure 4.1: Definition of the inertial system frame according to stapes geometry.

The nine coordinates were the following:

- Six in order to describe the rigid solid motion of the MIC: x_{cg}^{MIC}, y_{cg}^{MIC}, z_{cg}^{MIC}, α_{cg}^{MIC}, β_{cg}^{MIC}, γ_{cg}^{MIC}. These coordinates represented the motion of local reference system attached to the MIC and placed in its center of gravity. So, they represented its motion as rigid solid and they are defined in the inertial system: three translations and three rotations expressed in Kardan angles.

- Three in order to describe the piston-like motion of the stapes and two rocking motion: y_{cg}^{S}, α_{cg}^{S}, γ_{cg}^{S}. As in the case of the MIC, y_{cg}^{S} represented the translation of the local reference system attached to the center of gravity of the stapes in the y axis. α_{cg}^{S} represented the rocking motion around the current x axis of the local referent system and γ_{cg}^{S} the rocking motion around the current z axis of the local referent system. These coordinates are defined in the inertial system, as well

Thus, the vector of generalized coordinates q was as it is shown in 4.1:

$$q = \begin{bmatrix} x_{cg}^{MIC} \\ y_{cg}^{MIC} \\ z_{cg}^{MIC} \\ \alpha_{cg}^{MIC} \\ \beta_{cg}^{MIC} \\ \gamma_{cg}^{MIC} \\ y_{cg}^{S} \\ \alpha_{cg}^{S} \\ \gamma_{cg}^{S} \end{bmatrix} \tag{4.1}$$

The vector of generalized forces was the correspondent to an excitation at the umbo. A graphical representation of the model is shown in the figure 4.2.

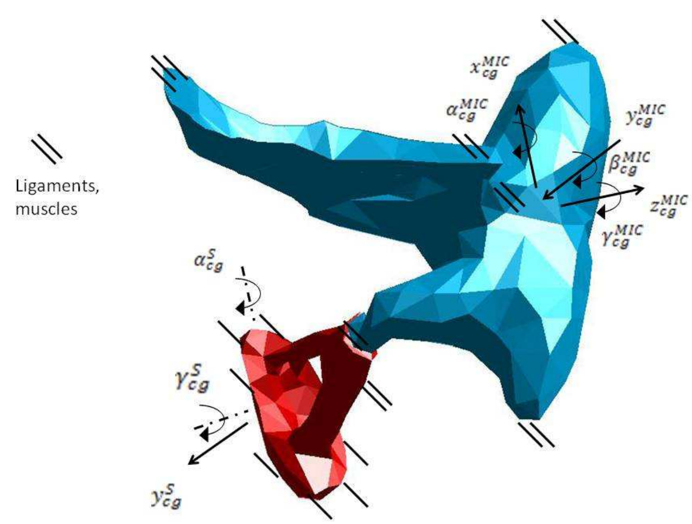


Figure 4.2: Mechanical model of the middle ear. The ligaments and muscles are indicated as black lines. In particular the translations x, y and z and rotations α, β and γ of the MIC (superscript MIC), translation y and rotations α and γ of the stapes (superscript S) are specified.

This model was implemented in SYMBS (Symbolic Multibody Systems), a softwared developed at the Institute of Engineering and Computational Mechanics. It is based on

Matlab's Symbolic Toolbox which allows symbolic algebraic computations within Matlab. SYMBS is able to derive the symbolic equations of motion of tree-structured holonomic multibody systems automatically, and further, a symbolic linearization of the equations of motion with respect to an arbitrary symbolic reference motion is provided.

4.2 Modeling of Bony Parts

The first step in order to model the ossicles was the estimation of their mechanical properties. However, the only available data about the GP were the CT data about their geometry. So, the approach was to build a finite element model of the ossicles in the commercial software ANSYS ®from this geometric data (see appendix *a.1.*).

The CT data were in stl format. ANSYS is not able to read this kind of files but since the stl file, in the ASCII format, had the information about the location of every point and the connectivity between them it was very simple to translate the CT data into an input file in a text format. However, due to the high number of points of the files, it was necessary to get a rougher mesh of the surface. So, the commercial software BLENDER®was used (see appendix *a.2.*) and, thus, the computer effort was significantly reduced. Anyway, the software BLENDER®generates binary stl files. Due to it was necessary the stl file in the ASCII format, the commercial software PROE®was used in order to convert the binary file into ASCII file.

In ANSYS®was necessary to close the holes of the malleus handle. This holes were not realistic and appeared in the original CT data because of precision problems since that zone of the bone was very porous. It was not only a graphical operation but also modeling operation since it needed to create a close surface. The reasons, besides to make a more realistic model, were the following:

- In order to estimate the mechanical properties it is necessary to create a volume to mesh, assign density properties, etc. to make the operations.
- In order to create a new stl file with no holes in the malleus handle it is necessary to have a close surface.

It was well-known that the bone structure of the ossicles was not homogenus. This structure consisted in an inner part of porous bone or trabecular bone and an outer part of more dense tissue or cortical bone there were no data available about the density values of this two parts neither their thickness. However, data about mechanical properties of the human middle ear were available (cite sim) as well as geometry data. Thus, FE models of the malleus and incus of human were built in ANSYS®in the same way than in GP case and the results in the homogeneous case were compared with data from literature (see table 4.1). In order to be able to compare these results the moments of inertia were all divided by their smaller component which in all the cases was the component of the principal axis in the superior-inferior direction.

Bone	Properties		Mean	Mean relative value	Model	Model relative value
Malleus	Mass $[mg]$		30.3	-	27.91	-
	Density $[mg/mm^3]$		2.39	-	2.64	-
	Principal inertia $[mg \cdot mm^2]$	n^M_{AP}	106.1	6.13	95.16	6.1
		n^M_{SI}	17.3	1 15.6		1
		n^M_{LM}	100.6	5.82	88.6	5.68
Incus	Mass $[mg]$		32.0	-	27.34	-
	Density $[mg/mm^3]$		2.15	-	2.47	-
	Principal inertia $[mg \cdot mm^2]$	n^I_{AP}	59.5	1.68	44.59	1.82
		n^I_{SI}	35.3	1	24.5	1
		n^I_{LM}	84.3	2.39	62.5	2.55

Table 4.1: Comparison of the principal moments of inertia. n^M_{AP}, n^I_{AP}: Principal axis in the anterior-posterior direction. n^M_{SI}, n^I_{SI}: Principal axis in the superior-inferior direction. n^M_{LM}, n^I_{LM}: Principal axis in the lateral-medial direction

The maximum committed relative error was 2% in the case of the malleus and 7% in the case of the incus. That meant that the approach was reasonable. So, a sensitive analysis was carried out in order to estimate the influence of the cortical bone thickness in the mechanical properties. This thickness was estimated as it is shown in equation 4.2. Table number (4.2) shows the different simulations played. As it is possible to see, the cortical bone thickness did not have pretty much influence in the mechanical properties. Therefore, it was assumed that this thickness was around the 10% of the volum of the bone.

$$thickness(mm) = thickness_{estimation}(\%) \cdot \sqrt[3]{volume(mm^3)} \tag{4.2}$$

This estimation of the thickness was played in the GP ossicles and obtained their me-

Bone	Properties		Simulations			
			a	b	c	d
Malleus	Density (cort. /trab.) $[mg/mm^3]$		2.8/1.5	2.8	2.8/1.5	2.8/1.0
	Thickness of trab. bone [%]		5	-	10	5
	Principal inertia $[mg \cdot mm^2]$	n^M_{AP}	5.91	6.1	5.98	5.97
		n^M_{SI}	1	1 1	1	1
		n^M_{LM}	5.5	5.68	5.57	5.55
Incus	Density (cort. /trab.) $[mg/mm^3]$		2.8/1.5	2.8	2.8/1.5	2.8/1.0
	Thickness of trab. bone [%]		5	-	10	5
	Principal inertia $[mg \cdot mm^2]$	n^I_{AP}	1.83	1.82	1.82	1.82
		n^I_{SI}	1	1	1	1
		n^I_{LM}	2.54	2.55	2.54	2.54

Table 4.2: Sensibility analisis of the cortical bone thickness influence. n^M_{AP}, n^I_{AP}: Principal axis in the anterior-posterior direction. n^M_{SI}, n^I_{SI}: Principal axis in the superior-inferior direction. n^M_{LM}, n^I_{LM}: Principal axis in the lateral-medial direction

chanical properties (see table 4.3).

These FE models, with a rougher mesh (lower computational effort) and with their geometry fixed, were used to created new stl files to export to SIMBS and have a more realistic graphical animation. However, in the case of the MIC, some modifications in the geometry were made. These modifications were about the zone of the malleus handle. As it is possible to see in the figure (4.3) there is a part of the handle that is clearly different. This part is a bony tissue as well but it belongs to the tympanic membrane. Thus, although in the estimation of the inertial properties this part was taken in count, in the graphical representation was removed (see fig. 4.4).

As soon as the tympanic membrane part was removed, it was possible to see the I-beam like cross-sectional shape in the malleus handle, as it was mentioned in the previous studies [10] (see fig. 4.5).

Furthermore, other point to check how good the geometric MIC model is is the lever ratio in the hinge-like motion. This ratio is defined as the ratio of the vertical distance between the rotational axis and the umbo to the vertical distance between the rotational axis and the incus tip (L_1/L_2 in fig. 4.6). According to the literature [9] this ratio has, tipically, a value of 2.1. In this model, the lever ratio had a value of 2.045, a value which is in range.

These new geometries (MIC in fig. 4.4 as well as stapes in fig. 4.7) were possible to

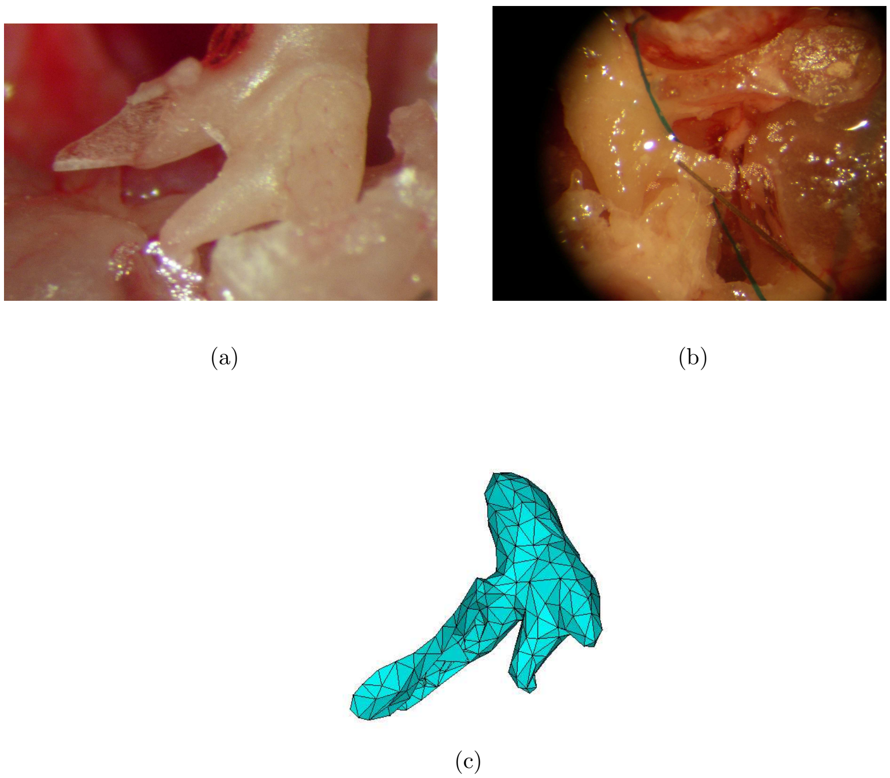


Figure 4.3: (a) Detail of the different kinds of bone in the malleus handle. (b) View of the tympanic membrane tissue attached to the MIC. (c) Model of the MIC including the tympanic membrane tissue.

Ossicles	Mechanical properties	
MIC	Mass (Kg)	$5.738 \cdot 10^{-6}$
	Ixx $(Kg \cdot m^2)$	$2.726 \cdot 10^{-11}$
	Ixy (")	$-1.246 \cdot 10^{-11}$
	Ixz (")	$5.68 \cdot 10^{-12}$
	Iyy (")	$1.072 \cdot 10^{-11}$
	Iyz (")	$-3.41 \cdot 10^{-12}$
	Izz (")	$7.35 \cdot 10^{-12}$
Stapes	Mass (Kg)	$4.958 \cdot 10^{-6}$
	Ixx $(Kg \cdot m^2)$	$1.122 \cdot 10^{-13}$
	Ixy (")	0
	Ixz (")	0
	Iyy (")	$1.004 \cdot 10^{-13}$
	Iyz (")	0
	Izz (")	$9.26 \cdot 10^{-14}$

Table 4.3: Masses and inertial tensors of MIC and stapes. Each inertial tensor was calculated with regard to the frame located in the center of gravity of its body

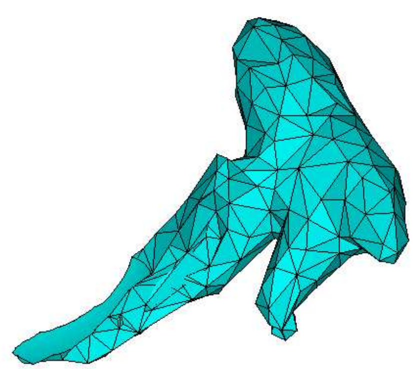


Figure 4.4: MIC model without the tympanic membrane.

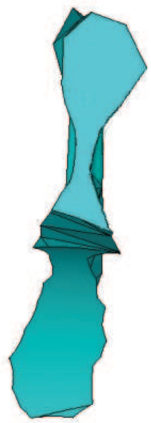


Figure 4.5: View of the malleus handle section.

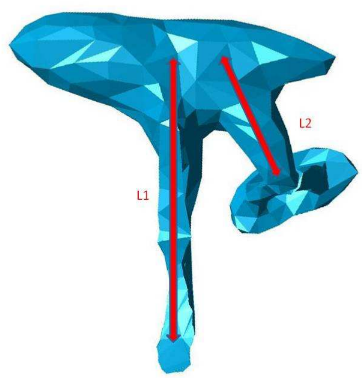


Figure 4.6: Lever ratio of the hinge-like motion of the MIC. Lever ratio: L_1/L_2).

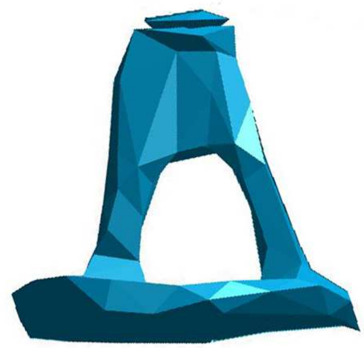


Figure 4.7: Model of the stapes.

export from ANSYS®, as IGES files, to PROE®in order to create new ASCII stl files. Finally, these new files were possible to pass as input files in the software SYMBS.

4.3 Modeling of Ligaments

As it was mentioned above, the joint, ligaments and muscles were modeled as visco-elastic elements. These elements are force elements which acts between two points described by two local coordinate systems. In the case of ligaments and muscles, one local coordinate system was attached to the ossicles and the other one was attached to the inertial system, which represented the tympanic cavity. In the case of the joint, one local coordinate system was attached to the MIC and the other one was attached to the stapes. It was assumed that two local reference systems belong to the same force element were placed, in the initial state, in the same point and with the same orientation according with the inertial system. In the most of the cases stiffness matrix only has non null elements in the diagonal. However, in the case of the lig. annulare and the tympanic membrane, due to they are considered as 3-dimensional elements, they have non null elements in the rest of the matrix components.

The stiffness and damping coefficients were passed as matrix or vector alternatively. With these stiffness and damping matrices, the force and moment was calculated by

$$\begin{bmatrix} \mathbf{F} \\ \mathbf{M} \end{bmatrix} = \mathbf{K} \cdot \begin{bmatrix} \mathbf{r} \\ \varphi \end{bmatrix} + \mathbf{C} \cdot \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\varphi} \end{bmatrix} , \tag{4.3}$$

where $\mathbf{r}$ and φ were the relative position and orientation between the two coordinate systems. In case that the coefficients were passed as vectors, the force and moment was calculated as

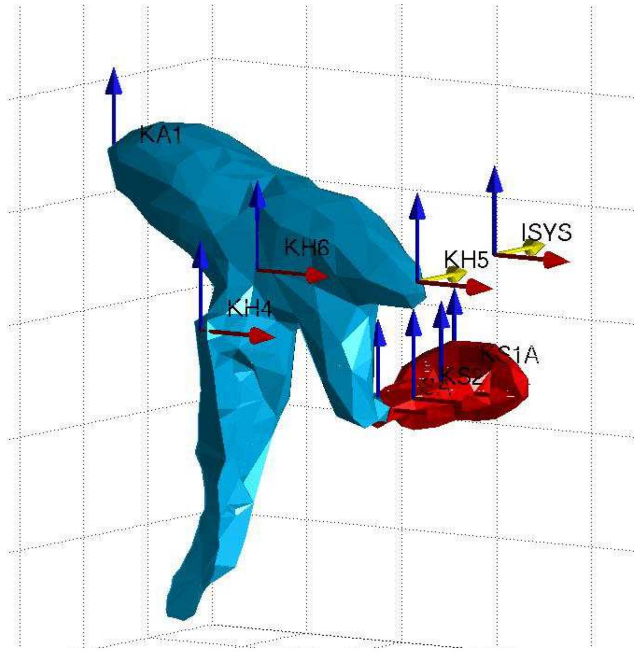
$$\begin{aligned} F(j) &= k(j)r(j) + c(j)\dot{r}(j) \ , \quad j = 1 \div 3 \ , \\ M(j) &= k(j+3)\varphi(j) + c(j+3)\dot{\varphi}(j) \ , \quad j = 1 \div 3 \end{aligned} \tag{4.4}$$

Attachment points were estimated according to literature data and using a graphical representation of the bones. Table 4.4 identifies the different ligaments and muscles, whose attachment points are shown in the figure 4.8. Since the attachment point of the tympanic membrane was the same of the application force point, it will be seen more clearly in the figure 4.9.

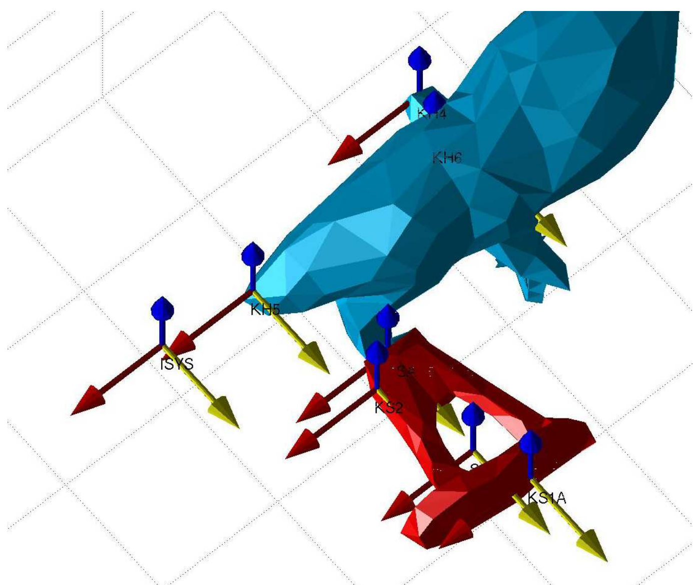
Ligaments and muscles markers	
Name	Marker
Musculus tensor tympani	FH4
Lig. mall. anterior	FH5
Lig. mall. laterale	FH6
Lig. inc. posterior	FA1
Incudo-stapedial joint	FAS
Musculus stapedius	FS2
Lig. annulare	FAR
Tympanic membrane	FAT

Table 4.4: Markers of the differents ligaments, muscles and joints modeled.

Due to there are no data available, human data (ref.) were taken to estimate the stiffness and damping properties of the ligaments and muscles. These human data were scaled by a factor which was estimated according to the equation 4.5:



(a)



(b)

Figure 4.8: Attachment points to the ossicles of the differents ligaments and muscles.

$$\frac{c_1}{c_2} = \frac{\omega_1^2}{\omega_2^2} \cdot \frac{m_1}{m_2} \tag{4.5}$$

Where c is stiffness, ω is the first eigenfrequency and m the mass; $\{\}_1$ are human values and $\{\}_2$ are GP values. In order to estimate this ratio a linear mass oscillator was assumed, with a rotational motion around the axe that is made by the points $KH5$ and $KA1$ (see fig. 4.8. This assumption makes sense since, how it will explained later, the first eigenmode consists in a piston-like motion of the stapes and a hinge-like motion of the MIC around this axe. So, typical values of the first eigenfrequencies were taken and inertial values were estimated with mass and distance values (see table 4.5).

	1 Frequency	$I_{Tim.membrane}$	I_{stapes}	$I_{mall.handle}$	I_{Incus}
Human	1000	$6.44 \cdot 10^{-10}$	$2.4 \cdot 10^{-10}$	$1.625 \cdot 10^{-10}$	$2.7 \cdot 10^{-11}$
GP	2000	$2.54 \cdot 10^{-11}$	$12.37 \cdot 10^{-12}$	$8 \cdot 10^{-12}$	$2 \cdot 10^{-13}$

Table 4.5: Numerical values of first eigenfrequencies and inertial moments of tympanic membrane, malleus handle, stapes and incus in both human and GP cases. All the values in I.S. units: Hz and $Kg \cdot m^2$.

With these values a scale factor of 6 were got. Thus, the initial values of these parameters are shown in the table 4.6.

4.4 External Forces

As it was mentioned before it was assumed in this model that the excitation was played as a force applied in the umbo. This force only had a y component (see fig. 4.9) and it was an harmonic function in the form:

$$f(t) = asin(2\pi ft) \tag{4.6}$$

The amplitude of the excitation was estimated knowing that, while in the human case the usual level of noise in the tympanic membrane is about 60 dB, in the GP the usual level of noise in the tympanic membrane is around 94 dB. According to this, and taking the value of the tympanic membrane area from [5] ($A_{tym.membrane} = 23.9mm^2$) it was possible

Ligaments and muscles parameters				
Element	Stiffness component	Value	Damping component	Value
Musculus tensor tympani	CH4X	5	DH4X	$5 \cdot 10^{-4}$
	CH4Y	11.66	DH4Y	$8.33 \cdot 10^{-4}$
	CH4Z	6.66	DH4Z	$5 \cdot 10^{-4}$
	CH4AL	0	DH4AL	0
	CH4BE	0	DH4BE	0
	CH4GA	0	DH4GA	0
Lig. mall. anterior	CH5X	1083	DH5X	$25 \cdot 10^{-3}$
	CH5Y	216.6	DH5Y	$11.6 \cdot 10^{-3}$
	CH5Z	216.6	DH5Z	$11.6 \cdot 10^{-3}$
	CH5AL	$12.8 \cdot 10^{-4}$	DH5AL	0
	CH5BE	$12.8 \cdot 10^{-5}$	DH5BE	0
	CH5GA	$12.8 \cdot 10^{-5}$	DH5GA	0
Lig. mall. laterale	CH6X	50	DH6X	$5 \cdot 10^{-4}$
	CH6Y	133.3	DH6Y	$1.66 \cdot 10^{-3}$
	CH6Z	50	DH6Z	$5 \cdot 10^{-4}$
	CH6AL	0	DH6AL	0
	CH6BE	0	DH6BE	0
	CH6GA	0	DH6GA	0
Tympanic membrane	KT11	$591.6 \cdot 10^{-1}$	DTX	$1.66 \cdot 10^{-3}$
	KTXY	$20.3 \cdot 10^{-1}$	DTY	0
	KTXZ	-0.18	DTZ	$1.66 \cdot 10^{-3}$
	KTXα	$-0.32 \cdot 10^{-3}$	DTAL	0
	KTXβ	$1.29 \cdot 10^{-1}$	DTBE	0
	KTXγ	$0.56 \cdot 10^{-1}$	DTGA	0
	KTYY	$73.5 \cdot 10^{-1}$	-	-
	KTYZ	$-61.5 \cdot 10^{-1}$		
	KTα	$0.25 \cdot 10^{-2}$		
	KTYβ	$-0.24 \cdot 10^{-4}$		
	KTYγ	$-0.928 \cdot 10^{-6}$		
	KTZZ	$720 \cdot 10^{-1}$		
	KTZα	$-0.585 \cdot 10^{-1}$		
	KTZβ	$9.72 \cdot 10^{-4}$		
	KTZγ	$-1.33 \cdot 10^{-5}$		
	KT$\alpha\alpha$	$2.6 \cdot 10^{-4}$		
	KT$\alpha\beta$	$-1.8 \cdot 10^{-4}$		
	KT$\alpha\gamma$	$-3.98 \cdot 10^{-8}$		
	KT$\beta\beta$	$4.18 \cdot 10^{-4}$		
	KT$\beta\gamma$	$1.8 \cdot 10^{-4}$		
	KT$\gamma\gamma$	$7.81 \cdot 10^{-5}$		
Lig. inc. posterior	CA1X	666	DA1X	$3.33 \cdot 10^{-3}$
	CA1Y	83.3	DA1Y	$3.33 \cdot 10^{-3}$
	CA1Z	50	DA1Z	$3.33 \cdot 10^{-3}$
	CA1AL	$3.83 \cdot 10^{-5}$	DA1AL	0
	CA1BE	$3.83 \cdot 10^{-5}$	DA1BE	0
	CA1GA	$3.83 \cdot 10^{-5}$	DA1GA	0

Lig. inc. posterior	CA1X	666	DA1X	$3.33 \cdot 10^{-3}$
	CA1Y	83.3	DA1Y	$3.33 \cdot 10^{-3}$
	CA1Z	50	DA1Z	$3.33 \cdot 10^{-3}$
	CA1AL	$3.83 \cdot 10^{-5}$	DA1AL	0
	CA1BE	$3.83 \cdot 10^{-5}$	DA1BE	0
	CA1GA	$3.83 \cdot 10^{-5}$	DA1GA	0
Incudo-stapedial joint	CASX	333.3	DASX	$8.33 \cdot 10^{-4}$
	CASY	333.3	DASY	$8.33 \cdot 10^{-4}$
	CASZ	333.3	DASZ	$1.66 \cdot 10^{-3}$
	CASAL	$4.16 \cdot 10^{-5}$	DASAL	$1 \cdot 10^{-9}$
	CASBE	$4.16 \cdot 10^{-5}$	DASBE	$5 \cdot 10^{-9}$
	CASGA	$4.16 \cdot 10^{-5}$	DASGA	$5 \cdot 10^{-9}$
Lig. annulare	KRYY	$5 \cdot 10^{1}$		
	KRYα	$-1.51 \cdot 10^{-5}$		
	KRYγ	$-1.57 \cdot 10^{-1}$		
	KRαY	$-2.03 \cdot 10^{-5}$		
	KR$\alpha\alpha$	$1.125 \cdot 10^{-5}$	-	-
	KR$\alpha\gamma$	$-1.585 \cdot 10^{-6}$		
	KRγY	$-0.16 \cdot 10^{-1}$		
	KR$\gamma\alpha$	$-1.553 \cdot 10^{-6}$		
	KR$\gamma\gamma$	$3.33 \cdot 10^{-5}$		
Musculus stapedius	CS2X	25	DS2X	$8.33 \cdot 10^{-4}$
	CS2Y	0	DS2Y	0
	CS2Z	0	DS2Z	0
	CS2AL	0	DS2AL	0
	CS2BE	0	DS2BE	0
	CS2GA	0	DS2GA	0

Table 4.6: Numerical values of the initial stiffness and damping coefficients (All the units in the I.S. x, y and z components are numerical values traslational spring stiffnesses in these directions while α, β and γ components are numerical values of rotational spring stiffnesses around the current x, y and z axes, respectively. In the case of Lig. annulare and tympanic membrane, there are the matrix components.

to calculate the amplitude which value is shown in the table 4.7. The frequency of the excitation was assumed equal than in the human case.

Excitation parameters	
Parameter	Value
Amplitude a (N)	$0.748 \cdot 10^{-6}$
Frequency f (Hz)	1000

Table 4.7: Numerical values of excitation force parameters.

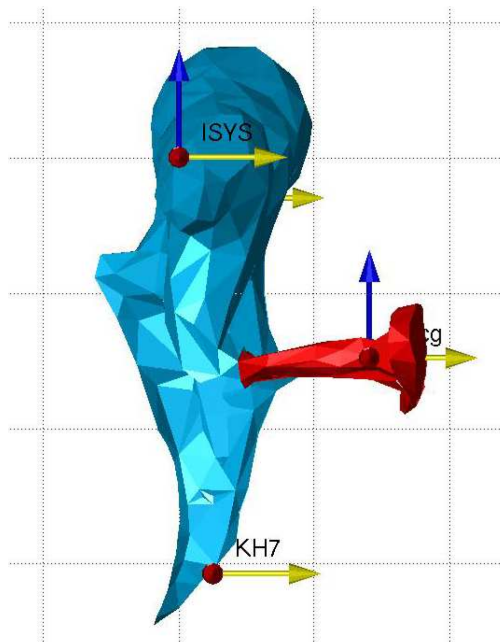


Figure 4.9: Application point of the excitation, *KH7*.